AN OPTIMAL REGULATOR FOR A SYNCHRONOUS GENERATOR

V. K. Bhan

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Vinod Kumar Bhan

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CERTIFICATE

Certified that this work on 'An Optimal Regulator for a Synchronous Generator' has been carried out under our supervision and that this has not been submitted elsewhere for a degree.

I G Sarma

Dr. I.G. Sarma
Assistant Professor
Elect. Engg. Department
IIT/Kanpur

M Ramamoorty

Dr. M.Ramamoorty
Assistant Professor
Elect. Engg. Department
IIT/Kanpur

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LIST OF SYMBOLS

d, q	direct and quadrature axes of the synchronous generator
ψ_d, ψ_q	d and q axis flux linkages
ψ_{kd}, ψ_{kq}	d and q axis amortisseur flux linkages
ψ_{fd}	field flux linkage
ψ_{ad}, ψ_{aq}	d and q axis mutual flux linkages
e_d, e_q, i_d, i_q	d and q axis voltage and current
v_{fd}, i_{fd}	field circuit voltage and current
i_{kd}, i_{kq}	d and q amortisseur circuit current
v	reference bus voltage
e_t	machine terminal voltage
e'	sending bus voltage
E_{fd}	air gap line open circuit excitation voltage
$x_{ffd}, x_{kkd}, x_{kkq}$	rotor circuit self reactances on d and q axes
x_{ad}, x_{aq}	mutual reactances on d and q axes
x_{f1}, x_{kd1}, x_{kq1}	leakage reactance of rotor circuit self reactances on d and q axes
x_{a1}	leakage reactance of mutual reactances on d and q axes
r_{fd}, r_{kd}, r_{kq}	rotor circuit resistance on d and q axes
r	armature resistance
Z_e, Z_t, Y_e	per phase parameters of terminal network

$X_e = wL_e$, $X_t = wL_t$, $B_e = wC_e$	reactance components of Z_e, Z_t, Y_e for steady sinusoidal operation
R_e, R_t, G_e	resistance components of Z_e, Z_t, Y_e
w_o, w	rated and instantaneous angular frequency in electrical rad/sec.
k_d, k_q	saturation factor for d and q axes
δ_1	rotor angle in radians
$\psi_{fdo}, \psi_{do}, \psi_{kdo}, \psi_{qo}, \psi_{kqo}, \delta_{10}, \delta_{20}, e_o$	operating point steady state value
K_d	damping factor
T_m	rotor inertia time constant in secs.
T_i, T_g	prime mover and generator air gap torque
Q	reactive generation at machine terminals
ΔT_1	change in torque input to rotor owing to governor action
V_r	voltage regulator reference voltage
V_s	derivative stabilizing signal
μ	voltage regulator open loop gain
μ_g	loop gain of prime mover governing system
μ_s	gain of derivative stabilizing loop
T_s	time constant of derivative stabilizing loop in secs.
T_h'	time constant representing turbine delay, in secs.

Δ	incremental operator
p	time differentiation operator
$\underline{A}$	system matrix without control
$\underline{B}$	input matrix
$\underline{C}$	output matrix
H	Hamiltonian
J	cost functional
$\underline{K}$	Riccati matrix

ABSTRACT

The modern control theoretic approach to the design of the Voltage Regulator and Speed Governor for a Synchronous Generator is given. An accurate linearized state space representation of the synchronous generator is formulated for which an optimal output regulator is designed. To implement this control a dynamical compatible observer which reconstructs the state vector is employed. The superiority in the performance of the system with optimal output feedback regulator compared to that with a conventional one is conclusively established.

CHAPTER I

INTRODUCTION

In recent years there has been a great deal of interest in the mathematical modeling of synchronous generators along with their controlling equipment mainly from the stand point of dynamic stability analysis, and the study of system performance during disturbance or abnormal conditions[6,7]; more recently there have also been attempts at the application of optimal control theory concepts to the design of the regulators[8].

In this thesis, a new approach to the design of the voltage regulator and the speed governor for the synchronous generator is given. This approach is diffe- rent from the conventional design procedure, wherein the designer chooses an a priori configuration for his controller and tries to adjust the various parameters to meet his requirements in so far as possible. In this approach an optimal feedback control law is first obtained for the synchronous generator which is then implemented by means of a dynamical compatible observer, the optimal control law is cascaded with the observer configuration resulting in the desired Optimal Output Regulator.

Chapter II deals with the mathematical representation of the synchronous generator in a form suitable for application of the techniques of modern control theory. The linearized state space model of the system developed, forms the basis for the investigations in the later chapters.

The conventional design procedure of the regulators is discussed in Chapter III and the performance of a generating system with a typical configuration of Voltage Regulator and Speed Governor is investigated.

In Chapter IV, an infinite time linear output regulator problem is formulated. The choice of this particular criterion is mainly dictated by the ease in the implementation of the resulting optimal controller. The optimal control law is obtained as the solution of the matrix Riccati equation, which has been solved by the technique of Successive Approximation[16], for different operating conditions and performance indicès. The implementation of the optimal control law requires the entire state vector for the purpose of feedback, a condition which is difficult to satisfy in many practical situations.

To obviate this difficulty an observer system can be employed, the design of which is treated in

Chapter V, it is shown that the observer system reconstructs the state vector from the available system inputs and outputs and the reconstructed state vector is used in the implementation of the optimal feedback control law. The performance of the Optimal Output feedback regulator is investigated and compared to that of the conventional regulator. The practical realization of the former is also indicated.

CHAPTER II

STATE SPACE MODEL OF THE SYNCHRONOUS GENERATOR

2.1 Introduction:

In the earlier studies[1-3], the mathematical representation of synchronous generator was based on the generalized theory of Kron or on the operational impedance approach of Park[4] and Concordia[5]. Recently, inspired by the developments in control system theory, Laughton[6], Undrill[7] and Yu etal[8] have extended the study of dynamical system behaviour. from the analysis of differential equation properties as exhibited by single higher order equations to the more general vector space methods.

In this chapter a state space model of the synchronous generator is formulated. The set of state variables chosen to describe the synchronous generator are the flux linkages of the direct and the quadrature axis armature and rotor circuits. These set of variables offer a definite advantage over the set of current and voltage variables chosen by Yu etal[8]. Since the flux linkages in a closed circuit cannot change suddenly, hence the initial conditions for any system disturbance can be found easily. The complexity in carrying out the analysis on the non-linear representation of the system is simplified by

linearization of the system differential equations about an operating point.

2.2 Mathematical Representation of Machine and Transmission Line:

A generating system, where the generator feeds a bus of known voltage and phase angle via a long transmission line with an additional shunt load at its sending end, as studied by Undrill[7], is chosen for investigation (see Fig.2.1).

A complete description of the dynamic behaviour of the synchronous generator requires consideration of its electrical and mechanical characteristics. The necessary mathematical statements as given by Park[4] in per unit and extensively used in literature are summarized in the following paragraphs. The assumptions made are:

1. The stator windings are sinusoidally distributed around the air gap as far as mutual effects between them and the rotor are concerned.

2. The stator winding self and mutual inductances vary sinusoidally as the rotor moves, and are of the form $a+b\cos 2\theta$, $a_1+b\cos 2(\theta + \frac{\pi}{3})$, respectively, where a, a_1 and b are constants.

3. Hysteresis effects are negligible.

4. Electrical transients on the transmission system are neglected.

5. Line parameters do not change with instantaneous generator frequency.

6. Leakage reactances of all the windings are independent of the saturation state of the iron.

7. The leakage fluxes do not contribute to the iron saturation, which is therefore determined by the mutual flux.

8. Mutual reactances between the direct axis armature winding and the two direct axis rotor circuits are made equal by suitable choice of the base rotor currents, and owing to the proximity of the amortisseur winding to the air gap, it is assumed that the mutual reactance between the two direct axis rotor circuits is equal to that between these circuits and armature.

9. Only those modes of operation that do not require zero axis variables are considered.

Electrical Equations:

A typical layout of the rotor field winding and stator armature windings of a 3-phase, 2 pole synchronous generator is shown in Fig.2.3. The path of the current flow in the rotor is represented by windings in direct and quadrature axis. The voltage and current equations of the synchronous generator are written for the individual circuits of the generator and mutual effect between them. These equations contain terms which are functions of rotor angle θ.

The difficulty in solution is overcome by Park's transformation. It resolves the stator quantities into components along the direct and quadrature axes. The resulting equations obtained for a balanced 3-phase performance are:

Direct axis flux linkage:

$$\psi_{fd} = x_{ffd} i_{fd} + x_{ad} i_{kd} - x_{ad} i_d \tag{2.2.1}$$

$$\psi_d = x_{ad} i_{fd} + x_{ad} i_{kd} - x_d i_d \tag{2.2.2}$$

$$\psi_{kd} = x_{ad} i_{fd} + x_{kkd} i_{kd} - x_{ad} i_d \tag{2.2.3}$$

Quadrature axis flux linkage:

$$\psi_q = x_{aq} i_{kq} - x_q i_q \tag{2.2.4}$$

$$\psi_{kq} = x_{kkq} i_{kq} - x_{aq} i_q \tag{2.2.5}$$

Direct axis voltage:

$$V_{fd} = \frac{1}{w_o} p\psi_{fd} + r_{fd} i_{fd} \tag{2.2.6}$$

$$e_d = \frac{1}{w_o} p\psi_d - r i_d - \frac{w}{w_o}\psi_q \tag{2.2.7}$$

$$0 = \frac{1}{w_o} p\psi_{kd} + r_{kd} i_{kd} \tag{2.2.8}$$

Quadrature axis voltage:

$$e_q = \frac{1}{w_o} p\psi_q - r i_q + \frac{w}{w_o}\psi_d \tag{2.2.9}$$

$$0 = \frac{1}{w_o} p\psi_{kq} + r_{kq} i_{kq} \tag{2.2.10}$$

Additional Equations:

Electrical torque in air gap:

$$T_g = \psi_d i_q - \psi_q i_d \tag{2.2.11}$$

Reactive power at terminals:

$$Q = e_q i_d - e_d i_q \tag{2.2.12}$$

Generator terminal voltage:

$$e_t^2 = e_d^2 + e_q^2 \tag{2.2.13}$$

Mechanical equation:

$$T_i = T_m(p^2\delta) + T_g + K_d(p\delta) + \Delta T \tag{2.2.14}$$

Terminal constraint equations:

The nodal equation for a T-equivalent of the transmission system, shown in Fig.2.2, is

$$\frac{e - e'}{Z_t} = e'Y_e + \frac{e' - v}{Z_e} \tag{2.2.15}$$

or $\quad i(Z_e+Z_tZ_eY_e+Z_t) = e(Y_eZ_e+1)-v$

Considering the d and q axes of the machine as the reference axes, the phase representation of terminal current and voltage is given by

$$i = i_d + ji_q$$

$$e = e_d + je_q$$

Substituting the values

$$(i_d+ji_q)((R_e+jX_e)+(R_t+jX_t)(R_e+jX_e)(G_e+jB_e)+(R_t+jX_t))$$

$$= (e_d+je_q)((R_e+jX_e)(G_e+jB_e)+1)-(v_d+jv_q)$$

Equating real and imaginary terms

$$\alpha e_d - \beta e_q = \eta i_d - \sigma i_q + v_d \tag{2.2.16}$$

$$\beta e_d + \alpha e_q = \sigma i_d + \eta i_q + v_q \tag{2.2.17}$$

where $\eta = R_e + R_t + R_eG_eR_t - (X_eB_eR_t+R_eB_eX_t+X_eG_eX_t)$

$$\sigma = X_e + X_t + R_eB_eR_t + X_eR_tG_e + R_eG_eX_t - X_eX_tB_e$$

$$\alpha = 1 + R_eG_e - X_eB_e$$

$$\beta = R_eB_e + X_eG_e$$

Rearranging the equations (2.2.16) and (2.2.17)

$$e_d = ei_d + fi_q + gv_d + hv_q \tag{2.2.18}$$

$$e_q = -fi_d + ei_q - hv_d + gv_q \tag{2.2.19}$$

where $$e = \frac{\eta\alpha + \sigma\beta}{\alpha^2+\beta^2}, \qquad f = \frac{\beta\eta - \alpha\sigma}{\alpha^2+\beta^2}$$

$$g = \frac{\alpha}{\alpha^2+\beta^2}, \qquad h = \frac{\beta}{\alpha^2+\beta^2}$$

To apply the techniques of modern control theory, it is convenient to represent the linear constant co-efficient

system in the form $\underline{\dot{x}} = \underline{A}\ \underline{x} + \underline{B}\ \underline{u}$, and the output equation written as $\underline{y} = \underline{C}\ \underline{x}$. The vector $\underline{u}$ comprises of the input variables to the system. The variables of vector $\underline{u}$ comprises ? of the input variables to the system. The variables of vector $\underline{y}$ are the available outputs of the system, which is being controlled. The vector $\underline{x}$ denotes the state of the system. The above set of equations (2.2.1) - (2.2.19) are manipulated to obtain a state space representation, the states being $\psi_{fd}, \psi_d, \psi_{kd}, \psi_{kq}, \delta_1, \delta_2$ and e_t, δ_1, δ_2 being the output variables.

Substituting the values of the currents in terms of the flux linkages in equations (2.2.1) - (2.2.19) results in

$$p\psi_{fd} = w_o\left(\frac{(\psi_{ad} - \psi_{fd})}{x_{f1}} r_{fd} + E_{fd}\frac{r_{fd}}{x_{ad}}\right) \tag{2.2.20}$$

$$p\psi_d = w_o\left(e_d + \psi_q \frac{w}{w_o} + \frac{r(\psi_{ad} - \psi_d)}{x_{a1}}\right) \tag{2.2.21}$$

$$p\psi_q = w_o\left(e_q - \psi_d \frac{w}{w_o} + \frac{r(\psi_{aq} - \psi_q)}{x_{a1}}\right) \tag{2.2.22}$$

$$p\psi_{kq} = \frac{w_o(\psi_{aq} - \psi_q)\ r_{kq}}{x_{kq1}} \tag{2.2.23}$$

$$p\psi_{kd} = -\frac{w_o\ r_{kd}\ (\psi_{ad} - \psi_{kd})}{x_{kd1}} \tag{2.2.24}$$

$$p\delta_1 = \delta_2 \tag{2.2.25}$$

$$p\delta_2 = \frac{1}{T_m}(T_i - T_g - K_d\delta_2 - \Delta T) \qquad (2.2.26)$$

$$e_d = \frac{e(\psi_{ad}-\psi_d)}{x_{a1}} + \frac{f(\psi_{aq}-\psi_q)}{x_{a1}} + gv\sin\delta_1 + hv\cos\delta_1 \qquad \ldots(2.2.27)$$

$$e_q = -\frac{f(\psi_{ad}-\psi_d)}{x_{a1}} + \frac{e(\psi_{aq}-\psi_q)}{x_{a1}} - hv\sin\delta_1 + gv\cos\delta_1 \qquad \ldots(2.2.28)$$

where $\psi_{ad} = \frac{1}{K_1}(\frac{\psi_d}{x_{a1}} + \frac{\psi_{fd}}{x_{f1}} + \frac{\psi_{kd}}{x_{a1}})$

$$\psi_{aq} = \frac{1}{K_2}(\frac{\psi_{kq}}{x_{kq1}} + \frac{\psi_q}{x_{a1}})$$

and $K_1 = \frac{1}{x_{ad}} + \frac{1}{x_{f1}} + \frac{1}{x_{k1}} + \frac{1}{x_{a1}}$

$$K_2 = \frac{1}{x_{aq}} + \frac{1}{x_{kq1}} + \frac{1}{x_{a1}}$$

Substitution of ψ_{aq}, ψ_{ad}, e_d and e_q in equations (2.2.20) - (2.2.28) results in

$$p\psi_{fd} = \frac{w_o r_{fd}}{x_{f1}}(\frac{1}{K_1 x_{f1}} - 1)\psi_{fd} + \frac{r_{fd} w_o}{K_1 x_{f1} x_{a1}}\psi_d + \frac{r_{fd} w_o}{K_1 x_{kd1} x_{f1}}\psi_{kd} + \frac{w_o r_{fd}}{x_{ad}}E_{fd} \qquad (2.2.29)$$

$$p\psi_d = \frac{w_o(r+e)}{K_1 x_{f1} x_{a1}}\psi_{fd} + \frac{w_o(r+e)}{x_{a1}}(\frac{1}{K_1 x_{a1}} - 1)\psi_d + \frac{w_o(r+e)}{K_1 x_{a1} x_{kd1}}\psi_{kd} + (w + \frac{fw_o}{x_{a1}}(\frac{1}{K_2 x_{a1}} - 1))\psi_q + \frac{fw_o}{K_2 x_{a1} x_{kq1}}\psi_{kq} + w_o gv\sin\delta_1 + w_o hv\cos\delta_1 \qquad (2.2.30)$$

$$p\psi_{kd} = \frac{w_o r_{kd}}{K_1 x_{kd1} x_{f1}}\psi_{fd} + \frac{w_o r_{kd}}{K_1 x_{kd1} x_{a1}}\psi_d + \frac{w_o r_{kd}}{x_{kd1}}\left(\frac{1}{K_1 x_{kd1}} - 1\right)\psi_{kd} \qquad \ldots(2.2.31)$$

$$p\psi_q = -\frac{fw_o}{K_1 x_{a1} x_{f1}}\psi_{fd} + \left(-w + \frac{w_o f}{x_{a1}}\left(1 - \frac{1}{K_1 x_{a1}}\right)\right)\psi_d - \frac{fw_o}{K_1 x_{a1} x_{kd1}}\psi_{kd}$$

$$+ \frac{w_o(r+e)}{x_{a1}}\left(\frac{1}{K_2 x_{a1}} - 1\right)\psi_q + \frac{w_o(r+e)}{K_2 x_{a1} x_{kq1}}\psi_{kq}$$

$$- w_o hv\operatorname{Sin}\delta_1 + w_o gv\operatorname{Cos}\delta_1 \qquad (2.2.32)$$

$$p\psi_{kq} = \frac{w_o r_{kq}}{K_2 x_{kq1} x_{a1}}\psi_q + \frac{w_o r_{kq}}{x_{kq1}}\left(\frac{1}{K_2 x_{kq1}} - 1\right)\psi_{kq} \qquad (2.2.33)$$

$$p\delta_1 = \delta_2 \qquad (2.2.34)$$

$$p\delta_2 = \frac{1}{T_m}\left(T_i - K_d\delta_2 - \frac{\psi_d\psi_q}{K_2 x_{a1}^2} - \frac{\psi_d\psi_{kd}}{K_2 x_{a1} x_{kq1}} + \frac{\psi_q\psi_d}{K_1 x_{a1}^2}\right.$$

$$\left. + \frac{\psi_q\psi_{fd}}{K_1 x_{a1} x_{f1}} + \frac{\psi_q\psi_{kd}}{K_1 x_{a1} x_{kd1}}\right) \qquad (2.2.35)$$

$$e_t^2 = e_d^2 + e_q^2 \qquad (2.2.36)$$

$$\delta_1 = \delta_1 \qquad (2.2.37)$$

$$\delta_2 = \delta_2 \qquad (2.2.38)$$

2.3 Linearized Model:

The system under consideration is basically a nonlinear system. Its performance is investigated by means

of linearized model. This form of analysis has been extensively used in literature[6-9]. It retains the first term of the Taylor series expansion of the equations about the operating point. The equations (2.2.29) - (2.2.38) are written for perturbed values of voltages and fluxes (such as $\psi_{do} + \Delta\psi_d$). Then subtraction of the original equations and neglecting of second order terms, yields the Linearized Model. The details of this are given in Appendix (A).

When linearized the system equations can be written in the following standard form

$$\begin{bmatrix} p\Delta\psi_{fd} \\ p\Delta\psi_d \\ p\Delta\psi_{kd} \\ p\Delta\psi_q \\ p\Delta\psi_{kq} \\ p\Delta\delta_1 \\ p\Delta\delta_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{46} & a_{45} & a_{46} & a_{47} \\ 0 & 0 & 0 & a_{54} & a_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & 0 & a_{77} \end{bmatrix} \begin{bmatrix} \Delta\psi_{fd} \\ \Delta\psi_d \\ \Delta\psi_{kd} \\ \Delta\psi_q \\ \Delta\psi_{kq} \\ \Delta\delta_1 \\ \Delta\delta_2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & b_{72} \end{bmatrix} \begin{bmatrix} \Delta E_{fd} \\ \Delta T_i \end{bmatrix} \qquad (2.3.1)$$

where $\Delta\psi_{fd}, \Delta\psi_d, \Delta\psi_{kd}, \Delta\psi_q, \Delta\psi_{kq}, \Delta\delta_1, \Delta\delta_2$ are the state variables of the system; and $\Delta E_{fd}, \Delta T_i$ are the forcing inputs to the system.

The output equations of the system are

$$\begin{bmatrix} \Delta e_t \\ \Delta \delta_1 \\ \Delta \delta_2 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta \psi_{fd} \\ \Delta \psi_d \\ \Delta \psi_{kd} \\ \Delta \psi_q \\ \Delta \psi_{kq} \\ \Delta \delta_1 \\ \Delta \delta_2 \end{bmatrix} \qquad (2.3.2)$$

where $\Delta e_t, \Delta\delta_1, \Delta\delta_2$ are the available outputs of the system.

$$a_{11} = \frac{w_o r_{fd}}{x_{f1}}\left(\frac{1}{K_1 x_{f1}} - 1\right) \qquad a_{12} = \frac{w_o r_{fd}}{K_1 x_{f1} x_{a1}}$$

$$a_{13} = \frac{w_o r_{fd}}{K_1 x_{kd1} x_{f1}} \qquad a_{21} = \frac{w_o (r+e)}{K_1 x_{f1} x_{a1}}$$

$$a_{22} = \frac{w_o (r+e)}{x_{a1}}\left(\frac{1}{K_1 x_{a1}} - 1\right) \qquad a_{23} = \frac{w_o (r+e)}{K_1 x_{a1} x_{kd1}}$$

$$a_{24} = w_o + \frac{f w_o}{x_{a1}}\left(\frac{1}{K_2 x_{a1}} - 1\right) \qquad a_{25} = \frac{f w_o}{K_2 x_{a1} x_{kq1}}$$

$$a_{26} = g w_o v \operatorname{Cos}\delta_{10} - h w_o v \operatorname{Sin}\delta_{10} \qquad a_{27} = \psi_{qo}$$

$$a_{31} = \frac{w_o r_{kd}}{K_1 x_{kd1} x_{f1}} \qquad a_{32} = \frac{w_o r_{kd}}{K_1 x_{kd1} x_{a1}}$$

$$a_{33} = \frac{w_o r_{kd}}{x_{kd1}}\left(\frac{1}{K_1 x_{kd1}} - 1\right) \qquad a_{41} = -\frac{f w_o}{K_1 x_{a1} x_{f1}}$$

$$a_{42} = -w_o + \frac{w_o f}{x_{a1}}\left(1 - \frac{1}{K_1 x_{a1}}\right) \qquad a_{43} = -\frac{f w_o}{K_1 x_{a1} x_{kd1}}$$

$$a_{44} = \frac{w_o (r+e)}{x_{a1}}\left(\frac{1}{K_2 x_{a1}} - 1\right) \qquad a_{45} = \frac{w_o (r+e)}{K_2 x_{a1} x_{kq1}}$$

$$a_{46} = -h w_o v \cos\delta_{10} - g w_o v \sin\delta_{10} \qquad a_{47} = -\psi_{do}$$

$$a_{54} = \frac{w_o r_{kq}}{K_2 x_{kq1} x_{a1}} \qquad a_{55} = \frac{w_o r_{kq}}{x_{kq1}}\left(\frac{1}{K_2 x_{kq1}} - 1\right)$$

$$a_{71} = \frac{\psi_{qo}}{T_m K_1 x_{a1} x_{f1}} \qquad a_{72} = -\frac{1}{T_m x_{a1}}\left(\frac{\psi_{qo}}{K_2 x_{a1}} + \frac{\psi_{kqo}}{K_2 x_{kq1}} - \frac{\psi_{qo}}{K_1 x_{a1}}\right)$$

$$a_{73} = \frac{\psi_{qo}}{T_m K_1 x_{a1} x_{kd1}} \qquad a_{74} = -\frac{1}{T_m x_{a1}}\left(\frac{\psi_{do}}{K_2 x_{a1}} - \frac{\psi_{do}}{K_1 x_{a1}} - \frac{\psi_{fdo}}{K_1 x_{f1}} - \frac{\psi_{kdo}}{K_1 x_{kd1}}\right)$$

$$a_{75} = -\frac{\psi_{do}}{T_m K_2 x_{a1} x_{kq1}} \qquad a_{77} = -\frac{K_d}{T_m}$$

$$b_{11} = \frac{w_o r_{fd}}{x_{ad}} \qquad b_{72} = \frac{1}{T_m}$$

$$c_{11} = \frac{e e_{do} - f e_{qo}}{e_o K_1 x_{f1} x_{a1}} \qquad c_{12} = \frac{1}{e_o x_{a1}}\left(\frac{1}{K_1 x_{a1}} - 1\right)(e e_{do} - f e_{qo})$$

$$c_{13} = \frac{ee_{do} - fe_{qo}}{K_1 x_{a1} x_{kd1} e_o} \qquad c_{14} = \frac{1}{e_o x_{a1}}\left(\frac{1}{K_2 x_{a1}} - 1\right)(fe_{do} + ee_{qo})$$

$$c_{15} = \frac{fe_{do} + ee_{qo}}{e_o K_2 x_{a1} x_{kq1}} \qquad c_{16} = \frac{1}{e_o}(v\cos\delta_{10}(ge_{do} - he_{qo}) - v\sin\delta_{10}(he_{do} + ge_{qo}))$$

2.4 System Performance:

For the purpose of analysis, a particular system with constants as given by Undrill[7] is taken and the relevant data are given below.

Machine Data

$x_{ad} = 1.0$ $\quad x_d = 1.2$ $\quad x_{aq} = 0.6$ $\quad x_{ffd} = 1.1$

$x_{kkq} = 0.8$ $\quad x_{kkd} = 1.1$ $\quad x_q = 0.8$ $\quad x_{a1} = 0.2$

$x_{f1} = 0.1$ $\quad x_{kd1} = 0.1$ $\quad x_{kq1} = 0.2$ $\quad r_{kd} = 0.02$

$r_{fd} = 0.0011$ $\quad r = 0.01$ $\quad r_{kq} = 0.04$ $\quad T_m = 6$ s

$K_d = 1.0$ $\quad w_o = 314.0$

Transmission Line Data

$R_e = 0.05$ $\quad R_t = 0.0$ $\quad X_e = 0.30$ $\quad G_e = 0.0$

$X_t = 0.0$ $\quad B_e = 0.0$

The operating point data calculated with an assumed saturation factor of unity are

$E_{fd} = 1.0$ $\qquad \psi_{do} = 0.5406$ $\qquad e_t = 0.8403$

$T_g = 0.6841$ $\delta_1 = 70.0°$ $\psi_{qo} = -0.6539$

$Q = -0.3276$ $v = 1.0$

These data are generally used in this thesis for the above mentioned operating conditions. The above system without any regulating equipment is simulated on a digital computer and its response for a perturbation in $\Delta\psi_{fd}$=0.05 p.u. is shown in Fig.2.4. Runge Kutta fourth order numerical integration technique (see Appendix (B)) is used for a step size of 0.0025. The step size chosen is smaller than the smallest time constant of the system. The response is rather poor, since even after the end of 5.0 seconds of real time, the outputs Δe_t and $\Delta\delta_1$ continue to rise, although the output $\Delta\delta_2$ tends to fall. The eigenvalues of this unregulated system are -0.046488, -0.26222, -0.63385, -27.826, -34.041, -71.616 $\pm$ j635.50. The negative real parts of the roots indicate that this system is stable.

2.5 Conclusion:

The state space model developed is an accurate representation of the system, since all the system parameters are included. This representation has an advantage over the other simplified models considered in literature. The simplified versions are usually obtained by omitting the terms $p\psi_d$ and $p\psi_q$ and sometimes the damper coils are assumed

nonexistant, resulting in the removal of rows and columns corresponding to ψ_{kd} and ψ_{kq} in the matrix representation.

The model is superior to that derived by Undrill[7], in the sense that no matrix inversion is involved. Further it is ideally suited for the direct application of modern control techniques. The inherent poor response of the system obviously suggests the incorporation of a regulator in the system to improve the response.

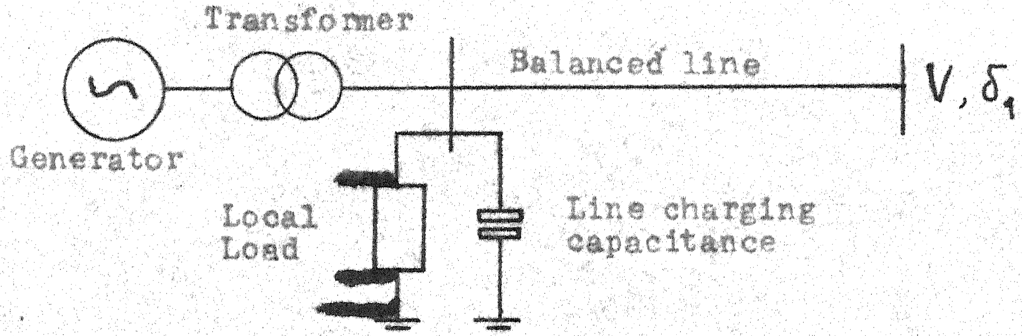


Fig.2.1 Single line diagram of a balanced 3-phase transmission line and load system.

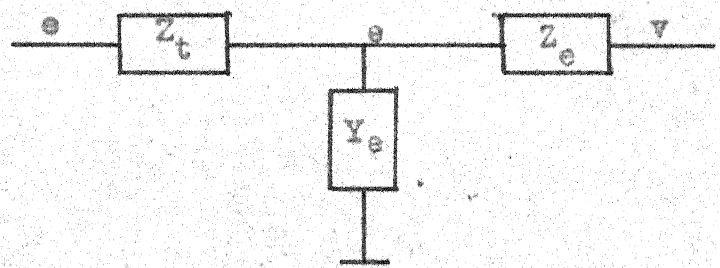


Fig.2.2 Per phase equivalent circuit for the transmission line shown in Fig.2.1.

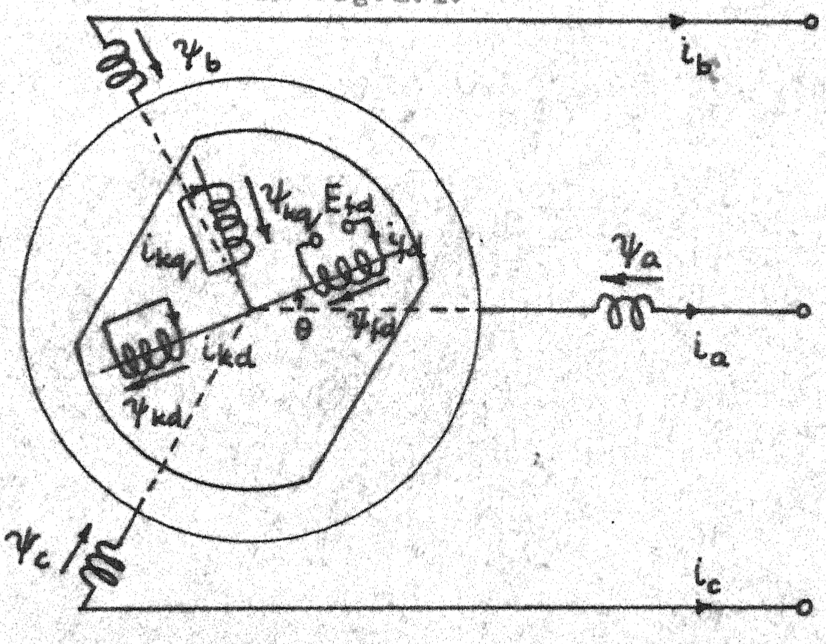


Fig.2.3 Schematic diagram of the Synchronous Generator.

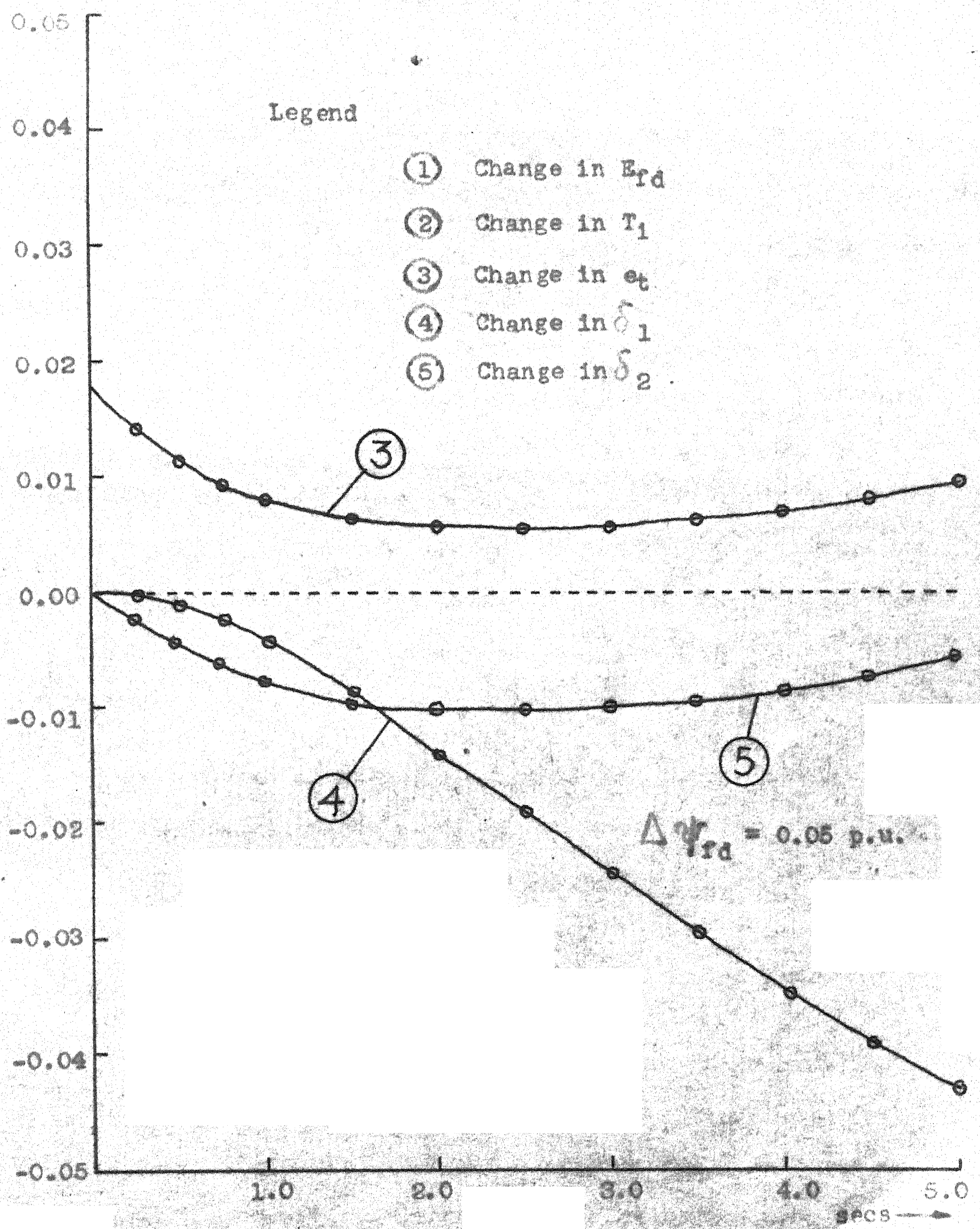


Fig. 2.4 Performance of the unregulated system.

CHAPTER III

PERFORMANCE WITH A CONVENTIONAL VOLTAGE REGULATOR AND SPEED GOVERNOR

3.1 Introduction:

Modern generating sets are generally operated under less stable conditions than in the past. This combined with the current trend towards lower inertia constants and higher transient reactances renders the generators more likely to become unstable during fault conditions. For a fixed excitation, there is a corresponding constant value of the power limit of the machine. By the use of continuously acting regulating equipment, usually regulating the terminal voltage and speed to some fixed reference, the transient stability limit of the generator can be improved[10].

In this chapter the conventional design procedure of regulating equipment is discussed. The performance of a generating system with a typical configuration of Voltage Regulator and Speed Governor is investigated.

3.2 Conventional Design Procedure:

The conventional design procedure starts with the selection of a suitable configuration of the regulating and stabilizing equipment. Typical configurations are given by Shackshaft[1], Meerov[11], Aldred[12] etc. The system analysis is

then carried out assuming the values for the time constants of the regulating and stabilizing equipment. The maximum overall gain of the loop is determined by the stability considerations, the most popular technique being the D-partition[13]. The gain is then distributed to the various components in the control loop depending upon their physical feasibility. The system response is then observed for its transient response and steady state error. Depending upon the response, the gains and time constants are further adjusted.

3.3 System Performance with Conventional Regulator:

A fairly common representation of a continuously acting Voltage Regulator with derivative stabilizing transformer and a conventional speed governor is the one used by Prabhashankar etal[14], shown in Fig.3.1 and 3.2. The control system equations are

Voltage Regulator:

$$pE_{fd} = \frac{1}{T_e}(\mu(V_r - e_t - V_s) - E_{fd}) \qquad (3.3.1)$$

$$pV_s = \frac{1}{T_s}(\mu_s(pE_{fd}) - V_s) \qquad (3.3.2)$$

Speed Governor

$$p(\Delta T_1) = \Delta T_2$$

$$p(\Delta T_2) = \frac{1}{T_g' T_h}\left(\frac{\mu_g \delta_2}{w_o} - (T_g' + T_h)\Delta T_2 - \Delta T_1\right) \qquad (3.3.4)$$

The above equations (3.3.1) - (3.3.4) are linearized and augumented to the system model, resulting in a state space representation of the form

$$\underline{\dot{x}} = \underline{W}\ \underline{x}$$

and $$\underline{y} = \underline{V}\ \underline{x}$$

where $\underline{W}$ is a 11x11 matrix and $\underline{V}$ is a 3x11 matrix, the state vector $\underline{x}$ is given by

$$\underline{x} = (\Delta\psi_{fd}\,\Delta\psi_d\,\Delta\psi_{kd}\,\Delta\psi_q\,\Delta\psi_{kq}\,\Delta\delta_1\,\Delta\delta_2\,\Delta E_{fd}\,\Delta V_s\,\Delta T_1\,\Delta T_2)^T$$

and the output vector $\underline{y}$ is given by

$$\underline{y} = (\Delta e_t\,\Delta\delta_1\,\Delta\delta_2)^T$$

The state space model of the system with the regulator is

$$\begin{bmatrix} p\Delta\psi_{fd} \\ p\Delta\psi_d \\ p\Delta\psi_{kd} \\ p\Delta\psi_q \\ p\Delta\psi_{kq} \\ p\Delta\delta_1 \\ p\Delta\delta_2 \\ p\Delta E_{fd} \\ p\Delta V_s \\ p\Delta T_1 \\ p\Delta T_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 & 0 & a_{18} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{54} & a_{55} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & 0 & a_{77} & 0 & 0 & a_{710} & 0 \\ a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & 0 & a_{88} & a_{89} & 0 & 0 \\ a_{91} & a_{92} & a_{93} & a_{94} & a_{95} & a_{96} & 0 & a_{98} & a_{99} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_{117} & 0 & 0 & a_{1110} & a_{1111} \end{bmatrix} \begin{bmatrix} \Delta\psi_{fd} \\ \Delta\psi_d \\ \Delta\psi_{kd} \\ \Delta\psi_q \\ \Delta\psi_{kq} \\ \Delta\delta_1 \\ \Delta\delta_2 \\ \Delta E_{fd} \\ \Delta V_s \\ \Delta T_1 \\ \Delta T_2 \end{bmatrix}$$

.... (3.3.6)

The output equations of the system is

$$\begin{bmatrix}\Delta e_t \\ \Delta\delta_1 \\ \Delta\delta_2\end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix}\Delta\psi_{fd} \\ \Delta\psi_d \\ \Delta\psi_{kd} \\ \Delta\psi_q \\ \Delta\psi_{kq} \\ \Delta\delta_1 \\ \Delta\delta_2 \\ \Delta E_{fd} \\ \Delta V_s \\ \Delta T_1 \\ \Delta T_2\end{bmatrix} \tag{3.3.7}$$

where

$$a_{18} = \frac{w_o r_{fd}}{x_{ad}} \qquad a_{710} = \frac{1}{T_m}$$

$$cc_1 = -\frac{\mu}{T_e} \qquad cc_2 = cc_1 \frac{\mu_s}{T_s}$$

$$a_{81} = cc_1 c_{11} \qquad a_{82} = cc_1 c_{12}$$

$$a_{83} = cc_1 c_{13} \qquad a_{84} = cc_1 c_{14}$$

$$a_{85} = cc_1 c_{15} \qquad a_{86} = cc_1 c_{16}$$

$$a_{88} = -\frac{1}{T_e} \qquad a_{89} = -\frac{\mu}{T_e}$$

$$a_{91} = cc_2 c_{11} \qquad a_{92} = cc_2 c_{12}$$

$$a_{93} = cc_2 c_{13} \qquad a_{94} = cc_2 c_{14}$$

$$a_{95} = cc_2 c_{15} \qquad a_{96} = cc_2 c_{16}$$

$$a_{97} = -\frac{\mu}{T_e T_s} \qquad a_{99} = -\frac{\mu\mu_s}{T_e T_s} - \frac{1}{T_s}$$

$$a_{117} = \frac{\mu_g}{w_o T_g' T_h} \qquad a_{1110} = -\frac{1}{T_g' T_h} \qquad a_{1111} = -\frac{(T_g' + T_h)}{T_g' T_h}$$

The other elements are the same as defined in section 2.3 .

For an initial perturnation in $\Delta\psi_{fd}$ = 0.05 p.u. the output response of the system with the above regulator is plotted in Fig.3.3. The regulator parameters are

$T_g' = 0.10$ $T_h = 0.50$ $\mu_s = 0.04$ $T_s = 1.00$

$T_e = 0.50$ $\mu = 5.00$ $\mu_g = 5.00$

These parameters are obtained by trial and error, so as to have a reasonable settling time and overshoot in the transient response of the system, for an initial perturbation in the state variables.

The oscillatory response of the system dies down in 10.0 seconds of the occurence of the initial perturbation in $\Delta\psi_{fd}$. The oscillatory response is due to the dominant pair of complex eigenvalues of the system. The variation in parameters of the regulator has a marked effect on the response of the system. An improper choice of the gains and the time constants can render the system unstable.

3.4 Conclusion:

The conventional regulator design has a drawback in the arbitrary choice of the regulator configuration and the cut and try procedure involved in the selection of the values of the parameters to meet the specifications such as overshoot, decay time and stability limits. Even though

these specifications are achieved, it may not represent the best possible design.

Through the choice of the appropriate performance index it becomes possible to impart the desired features to the system transient response. Furthermore, it becomes possible to explicitly take into account any constraints involving the control and output variables of the system. In all cases the optimal control law derived exploits the maximum number of 'degrees of freedom' (Horowitz) that are available. In the next chapter the design of an optimal regulator for the power system under consideration is presented.

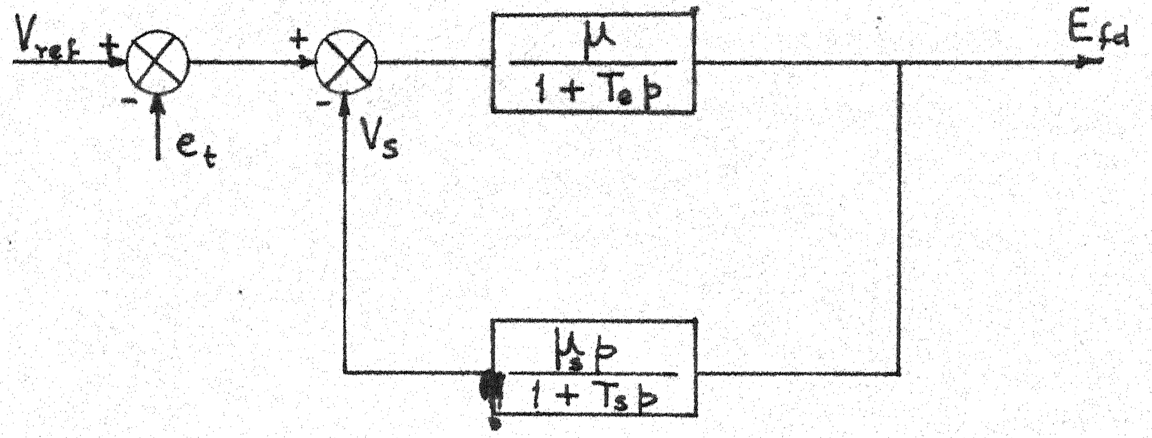


Fig. 3.1 Block diagram representation of a conventional voltage regulator.

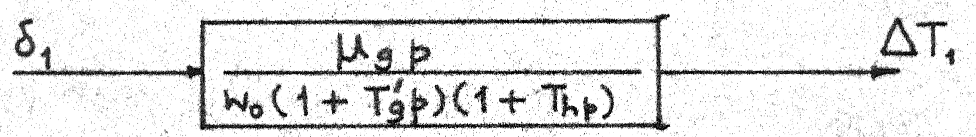


Fig.3.2 Block diagram representation of a speed governor.

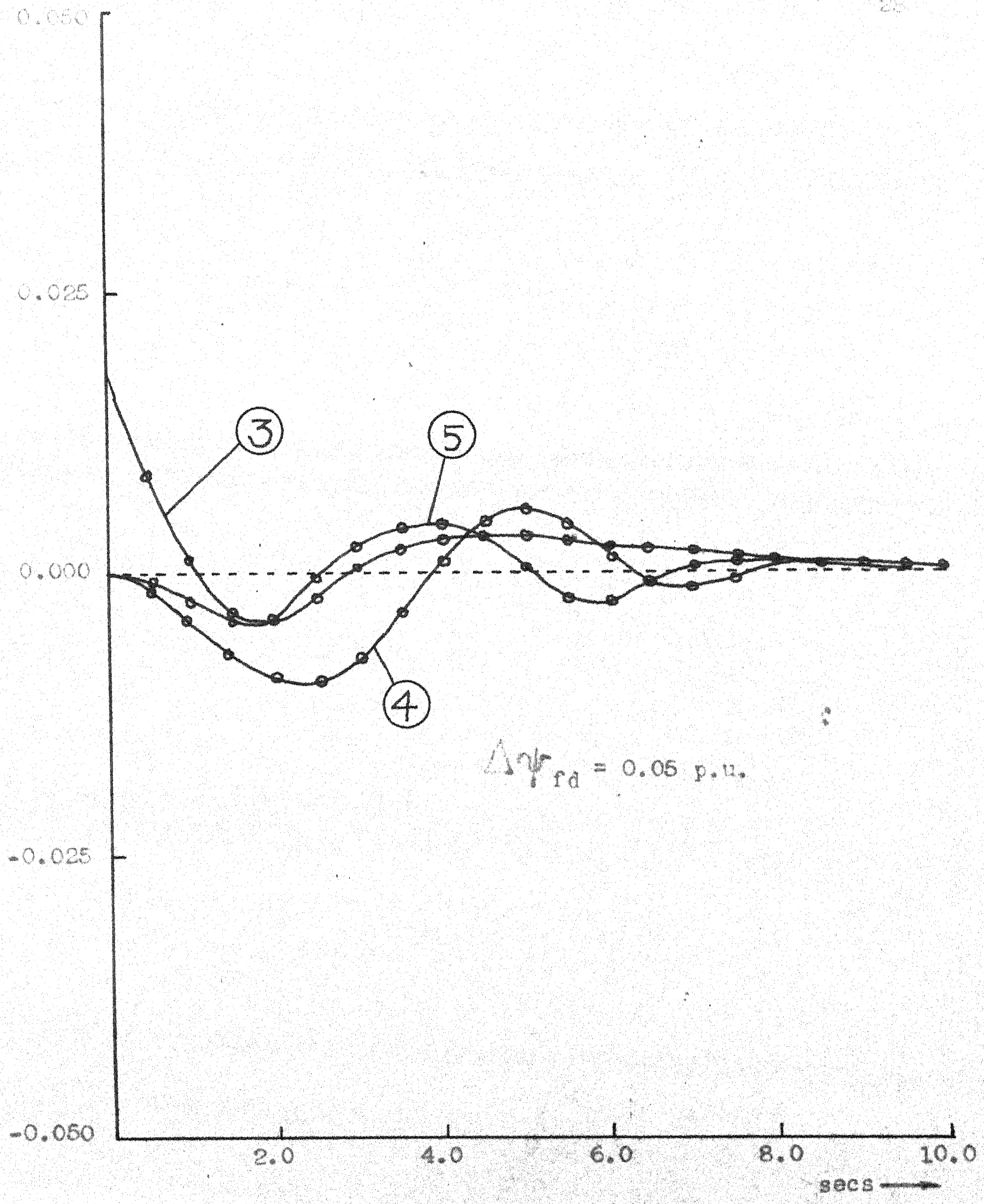


Fig.3.3 Performance of the system with a conventional regulator.

CHAPTER IV

DESIGN OF AN OPTIMUM VOLTAGE AND SPEED REGULATOR

4.1 Introduction

The design of a regulator for the synchronous generator, both for the control of the terminal voltage and speed, has been bestowed much attention in literature. Recently, Yu etal[8] have discussed an optimum output regulator for a synchronous generator system, already equipped with a voltage regulator and a speed governor. The output of the optimum controller is fed to two different summing junctions of the exciter voltage regulator and the speed governor through a transfer function and a servomotor respectively.

Thus, the problem considered by them is a "specific optimal" control problem, wherein the structure of one part of the controller comprising of a conventional voltage regulator and a speed governor is assumed to be fixed and the optimal controller which forms the remaining part of the controller is obtained by treating the synchronous generator with the voltage regulator and speed governor as the equivalent plant to be controlled. The inclusion of conventional regulators is superfluous as the optimum controller is itself capable of providing the necessary control. This latter approach is adopted in this investigation.

The system is formulated as an infinite time output regulator control problem. The theory of the infinite time regulator problem for a linear time-invariant plant with a quadratic objective functional, with no explicit constraints involving the control and the output variables is well known[15]. For this class of problems, the optimal controller obtained is linear and time-invariant and can be synthesised using the state vector feedback.

For a linear regulator problem, the optimal control law obtained requires the solution of a matrix Riccati equation. For higher order systems analytical solution to the resulting matrix Riccati equation becomes difficult. Therefore, the solution of the matrix Riccati equation is customarily obtained, by numerical techniques. The method of Successive Approximation[16] is fairly popular and is described in the following sections.

4.2 Formulation of the Optimum Control Problem:

In this section, the unregulated power system considered in Section 2.2 is formulated as an optimum output regulator problem[15]. The cost functional which is to be minimized is assumed to be of the form

$$J = \frac{1}{2}\int_0^{\infty} (\underline{y}^T \underline{Q}\, \underline{y} + \underline{u}^T \underline{R}\, \underline{u})\, dt \qquad (4.2.1)$$

where the matrix $\underline{Q}$ and the nonsingular matrix $\underline{R}$ are the

weightages associated with the output and the input variables and these are generally chosen to be symmetric matrices. The weightages assigned depends upon the relative importance of the variables in terms of the desired response. Since $\underline{u}$ and $\underline{y}$ are the deviations in control and output vectors, the minimization of the performance index results in minimizing the control effort as well as the error in the output response of the system after an initial disturbance.

The problem formulated above does not include any constraints of either the equality or the inequality type. Usually depending upon the physical nature of the system, certain variables are constrained. Optimal regulator problems with constraints are difficult to solve. The constraints can be indirectly taken into account by assigning suitable penalties to the constrained variables. The $\underline{Q}$ and $\underline{R}$ matrices themselves can be chosen to reflect these penalties.

Once the choice of the performance index is made, the optimal control law can be determined by the application of Pontryagin's maximum principle or the Hamilton-Jacobi theory. The optimal control $\underline{u}^*$ minimizes the objective functional given by the equation (4.2.1), and subject to the differential equality constraint

$$\underline{\dot{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u} \qquad (4.2.2)$$

which represents the state equation of the system and the algebraic equality constraint

$$\underline{y} = \underline{C}\ \underline{x} \tag{4.2.3}$$

which is the output equation of the system. Where $\underline{A}$ is a nxn matrix, $\underline{B}$ is a nxr matrix and $\underline{C}$ is a pxn matrix.

The existence of this $\underline{u}^*$ requires prior investigation before a formal solution to the above problem is attempted. It is ascertained by invoking the notion of output controllability. In the problem under consideration it is desired to transfer the system to its normal operating condition (origin for a linearized model) after an initial disturbance. The condition of controllability ensures that such a transfer of the system can be affected. The presence of uncontrollable dynamic modes implies that for this system, there does not exist any control which provides a transfer to the desired operating condition.

A system is said to be completely output controllable if it is possible to construct a control vector $\underline{u}(t)$ which will transfer any given initial output $\underline{y}(t_0)$ to any final desired output $\underline{y}(t_1)$ in a finite time interval $t_0 \leq t \leq t_1$. The condition for the system to be output controllable is that the matrix

$$\underline{P} = (\underline{C}\ \underline{B} \quad \underline{C}\ \underline{A}\ \underline{B} \quad \underline{C}\ \underline{A}^2\ \underline{B} \quad \ldots \quad \underline{C}\ \underline{A}^{n-1}\ \underline{B})$$

be of rank p.

To check the controllability of the system under consideration, the 3x4 matrix $\underline{P}$ is formed, and the determinants of different combinations of 3 columns of matrix $\underline{P}$ were found to be non-zero. Thus the rank of $\underline{P}$ is 3 thereby implying that the system is controllable. As a further verification of this result, the system has also been reduced to its normal form, and it was found that the input matrix has no rows which have all zero elements. This ensures that the system is indeed output controllable[17].

4.3 Derivation of the Optimal Control Law:

Since the theory of optimal output regulator is well known, only the major steps are presented below, for the system given by equation (4.2.2) and (4.2.3) and the performance index given by equation (4.2.1).

The Hamiltonian H is first defined as

$$H = \tfrac{1}{2}\,\underline{y}^T \underline{Q}\,\underline{y} + \tfrac{1}{2}\,\underline{u}^T \underline{R}\,\underline{u} + \underline{\lambda}^T(\underline{A}\,\underline{x} + \underline{B}\,\underline{u}) \qquad (4.3.1)$$

where $\underline{\lambda}$ is the Lagrange multiplier vector.

For an optimal control $\underline{u}^*$, $\underline{\lambda}$ and $\underline{x}$ are the solutions of

$$\dot{\underline{x}} = \frac{\partial H}{\partial \underline{\lambda}}, \qquad \dot{\underline{\lambda}} = -\frac{\partial H}{\partial \underline{x}}$$

and

$$\frac{\partial H}{\partial \underline{u}} = \underline{0} \qquad (4.3.2)$$

that is,

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u}$$

$$\dot{\underline{\lambda}} = -\underline{C}^T\,\underline{Q}\,\underline{C}\,\underline{x} - \underline{A}^T\,\underline{\lambda}$$

and $$\underline{R}\,\underline{u} + \underline{B}^T\underline{\lambda} = \underline{0} \tag{4.3.3}$$

Solving for $\underline{u}$,

$$\underline{u} = -\underline{R}^{-1}\,\underline{B}^T\,\underline{\lambda} \tag{4.3.4}$$

Assuming that the Lagrange multiplier $\underline{\lambda}$ and $\underline{x}$ are related by the matrix equation

$$\underline{\lambda} = \underline{K}\,\underline{x} \tag{4.3.5}$$

the optimal control becomes

$$\underline{u}^* = -\underline{R}^{-1}\,\underline{B}^T\,\underline{K}\,\underline{x} \tag{4.3.6}$$

In the infinite-time case, the symmetric matrix $\underline{K}$ is the solution of the nonlinear Riccati equation

$$-\underline{A}^T\,\underline{K} - \underline{K}\,\underline{A} + \underline{K}\,\underline{B}\,\underline{R}^{-1}\,\underline{B}^T\,\underline{K} - \underline{C}^T\,\underline{Q}\,\underline{C} = \underline{0} \tag{4.3.7}$$

The optimally controlled system dynamics are governed by the homogeneous state equation

$$\dot{\underline{x}} = (\underline{A} - \underline{B}\,\underline{R}^{-1}\,\underline{B}^T\,\underline{K})\,\underline{x} \tag{4.3.8}$$

From the above equation it is obvious that the system dynamics are altered, hence even an unstable system can be stabilized with the application of the optimal control.

4.4 Application of Successive Approximation for the Solution of the Riccati Equation:

The solution of the 7x7 matrix Riccati equation implies the solving of 28 simultaneous nonlinear equations. They are difficult to solve analytically and therefore the method of Successive Approximation as given by Puri and Gruver[16] is adopted for the solution. In this method the approximation in control policy space is combined with stability considerations from the Direct Method of Liapunov to generate a sequence of suboptimal control functions, which have a monotonic convergence. A control function is first chosen which represents a stable system. The Hamiltonian is then minimized to obtain the second approximation.

The Riccati equation at the k^{th} iteration is given by

$$\underline{K}^{(k)}\underline{A}^{(k)} + (\underline{A}^{(k)})^T \underline{K}^{(k)} + \underline{Q}^{(k)} = \underline{0} \qquad (4.4.1)$$

where $\underline{A}^{(k)} = \underline{A} - \underline{B}\,\underline{R}^{-1}\,\underline{B}^T\,\underline{K}^{(k-1)}$

and $\underline{Q}^{(k)} = \underline{C}^T\,\underline{Q}\,\underline{C} + \underline{K}^{(k-1)}\,\underline{B}\,\underline{R}^{-1}\underline{B}^T\,\underline{K}^{(k-1)}$

The above equation can be solved in a vector equation form

$$\underline{k}^{(k)} = -\underline{D}^{-1(k)}\,\underline{q}^{(k)} \qquad (4.4.2)$$

where $\underline{k}^{(k)} = (k_{11}^{(k)} \ldots k_{1n}^{(k)}\; k_{22}^{(k)} \ldots k_{2n}^{(k)} \ldots k_{nn}^{(k)})$,

$\frac{n(n+1)}{2}$ vectors

$$\underline{q}^{(k)} = (q_{11}^{(k)} \ldots q_{1n}^{(k)}\ q_{22}^{(k)} \ldots q_{2n}^{(k)} \ldots q_{nn}^{(k)}), \frac{n(n+1)}{2} \text{vectors}$$

$k_{ij}^{(k)}$ and $q_{ij}^{(k)}$ are the elements of $\underline{K}^{(k)}$ and $\underline{Q}^{(k)}$ and $\underline{D}^{(k)}$ is an $\frac{n(n+1)}{2} \times \frac{n(n+1)}{2}$ matrix whose elements are the combination of the a_{ij} of the matrix $\underline{A}^{(k)}$. The elements of $\underline{D}^{(k)}$ are obtained by equating the coefficients of equation (4.4.1) and is reduced to the following form

$$\underline{q}^{(k)} = -\underline{D}^{(k)}\ \underline{k}^{(k)} \qquad (4.4.3)$$

The solution of $\underline{k}^{(k)}$ vector results in the gain matrix $\underline{K}^{(k)}$.

The weighting matrices $\underline{Q}$ and $\underline{R}$ on the output and input vectors are chosen as $\underline{Q}$ = diag(100 100 100), $\underline{R}$ = diag(1 1). As the technique requires $\underline{K}^{(o)}$ to make the closed loop system stable, a choice of $\underline{K}^{(o)}$ is made and the new gain matrix $\underline{K}$ is then determined. The method of Successive Approximation is continued until the difference in the coefficients of matrix $\underline{K}$ between two consecutive iterations is less than 1%. The method converges in 10 iterations for the operating conditions given in Section 2.4. The matrices $\underline{B}, \underline{A}, \underline{C}$ for this operating conditions are

$$\underline{B} = \begin{bmatrix} 0.3454 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1667 \end{bmatrix}^T$$

$$\underline{A} = \begin{bmatrix} -2.1255 & 0.6642 & 1.3285 & 0 & 0 & 0 & 0 \\ 36.231 & -76.085 & 36.231 & 583.14 & -201.86 & 107.25 & -0.6540 \\ 24.154 & 12.077 & -38.646 & 0 & 0 & 0 & 0 \\ 181.15 & -694.42 & 181.15 & -53.829 & 40.371 & -295.12 & -0.5406 \\ 0 & 0 & 0 & 26.914 & -35.886 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -0.2096 & 0.3039 & -0.2096 & 0.3212 & -0.1931 & 0 & -0.1667 \end{bmatrix}$$

$$\underline{C} = \begin{bmatrix} 0.4460 & -0.9366 & 0.4460 & 0.5751 & -0.4313 & -0.3389 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and the matrix $\underline{K}$ found by the above technique is given by

$$\underline{K} = \begin{bmatrix} 5.2204 & -0.1364 & 0.3866 & -0.0125 & -0.2300 & 2.7671 & -4.3565 \\ -0.1364 & 0.4925 & -0.1455 & 0.0065 & 0.0013 & 0.7798 & -0.0298 \\ 0.3866 & -0.1458 & 0.0953 & -0.0088 & -0.0795 & -0.0124 & -0.4965 \\ -0.0125 & 0.0065 & -0.0088 & 0.3347 & -0.0866 & 0.0762 & 0.0328 \\ -0.2301 & 0.0013 & -0.0795 & -0.0866 & 0.1565 & -3.7236 & -0.4176 \\ 2.7671 & 0.7798 & -0.0124 & 0.0762 & -3.7236 & 269.17 & 91.553 \\ -4.3565 & -0.0298 & -0.4965 & 0.0328 & -0.4176 & 91.553 & 95.133 \end{bmatrix}$$

4.5 Effect of Different Performance Indices and Operating Points:

Different performance indices are generated by keeping the $\underline{R}$ matrix fixed as $\underline{R} = \text{diag}(1\ 1)$, and the weightages on the output vector sequentially being increased, the sequence being

$\underset{\sim}{Q}$ = diag(1 1 1), $\underset{\sim}{Q}$ = diag (10 10 10) and $\underset{\sim}{Q}$ = diag(100 100 100). The output response for these three cases is obtained by solving equations(4.3.8) and (4.2.3) for an initial perturbation in $\Delta\psi_{fd}$ = 0.05 p.u. is shown in Figs.4.1 - 4.3, the optimum control $\underset{\sim}{u}^*$ being determined from equation (4.3.6). It indicates a progressing increase in the control effort and the outputs have faster exponential decays in the increasing sequence For the last case the system response dies down within 2.5 seconds, although at the expense of extra control effort. The optimal control at two other operating conditions given in Table below

	Case (ii)	Case (iii)
δ_1	40°	10°
E_{fd}	1.0	1.0
v	1.0	1.0
ψ_{do}	0.9671	0.9956
ψ_{qo}	-0.2496	-0.1264
e_t	0.9990	1.0710

is determined. The response of the regulated system for an initial perturbation in $\Delta\psi_{fd}$ = 0.05 p.u. is shown in Figs. 4.4 and 4.5 reveals that the optimum control effort required is less as the load angle decreases. This ought to be expected since the restoring torque is more at smaller load angles.

4.6 Validity of the Linear Model:

The optimum control law obtained in equation (4.3.6) is for the linearized system model around a specific operating condition. The actual representation of the system model is nonlinear as indicated in Chapter II. Thus the validity of the optimum control derived on basis of the linearized model when applied to the actual nonlinear system remains to be investigated. The nonlinear model is represented by equations (2.2.29) to (2.2.38) to which the optimum controllaw given by the equation (4.3.6) is applied. As the optimum control law is represented in terms of the perturbed state vector, the latter is obtained by subtracting the actual value from the one corresponding to the steady state operating condition. The resultant output of the controller is added to the nominal control inputs corresponding to the steady state operating condition. This provides the total control to the nonlinear system.

The response for the operating condition given in Section 2.4 for the nonlinear model with the optimal control law was obtained for an initial perturbation in ψ_{fd}=0.05 p.u. is shown in Fig.4.6. The comparison of the response with the response obtained for the linearized model indicates, that the optimum control determined on the basis of the linearized model when implemented on the actual nonlinear

system gives quite satisfactory performance for small perturbation.

4.7 Conclusion:

The comparison of the response of the system with a conventional regulator (Fig.3.3 to an optimal regulator Fig.4.3) for the same perturbation in the variable $\Delta\psi_{fd}$ reveals the superiority of the latter. The output response decays exponentially, without any overshoot in the optimal case.

In the design of the Optimal Regulator the configuration of the controller is not assumed Yu etal[8] assume the configuration and have obtained a specific optimum control law. This is an unnecessary constraint on their design procedure.

The optimal control law obtained in equation (4.3.6) requires the entire state vector for the purpose of feedback. The states by themselves are not unique and depend upon the particular formulation of the state space model. Quite often these state variables are not available for direct measurement. On the other hand the system outputs are the ones which are available for control purposes. Therefore it becomes necessary that only outputs which are directly available should be used in the implementation of

the control law. This is carried out by reconstructing the state vector from the available system inputs and outputs and using the reconstructed state vector for the control purpose.

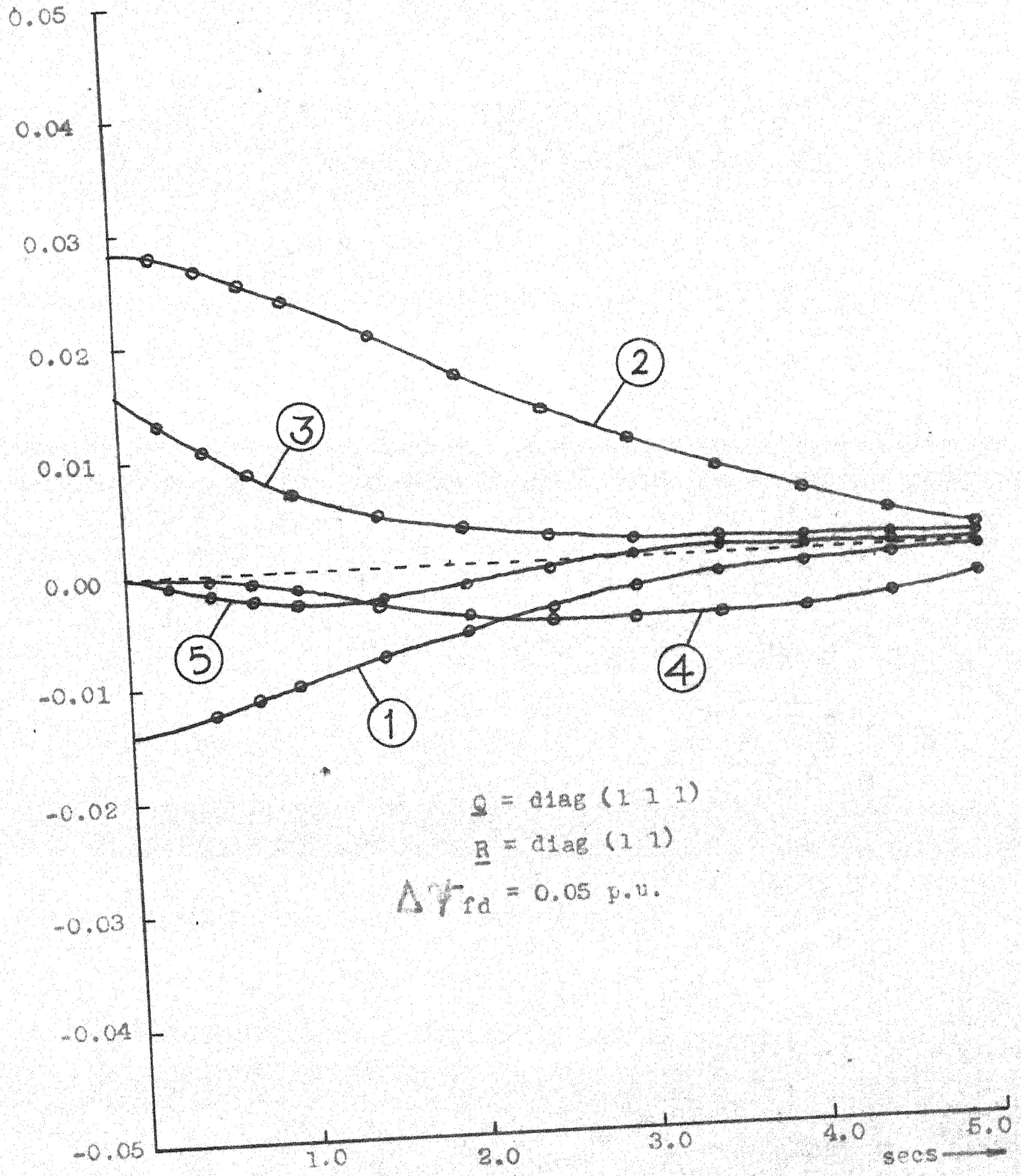


Fig.4.1 Performance of the state regulated optimal system for case (i).

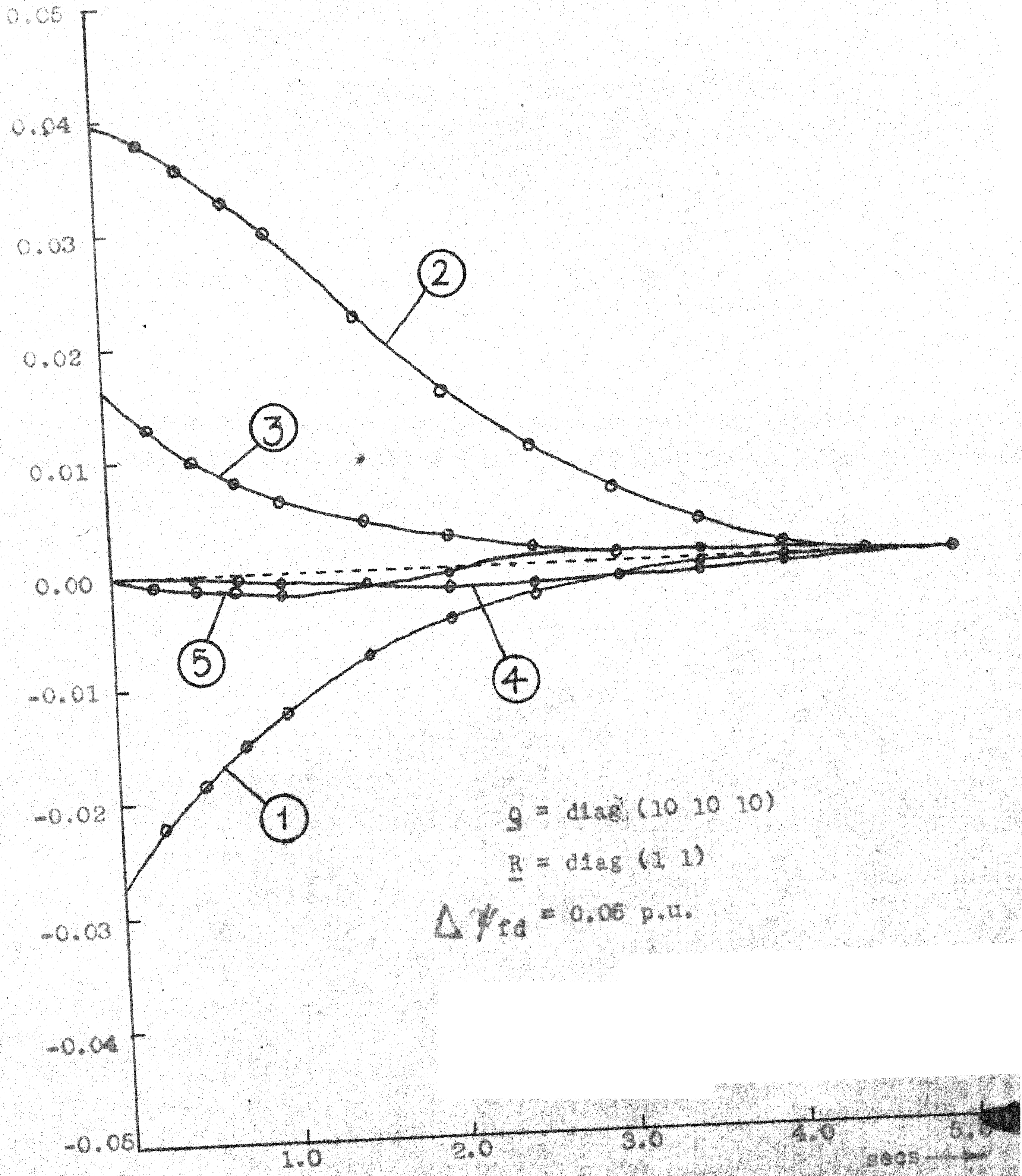


Fig.4.2 Performance of the state regulated optimal system for case (i).

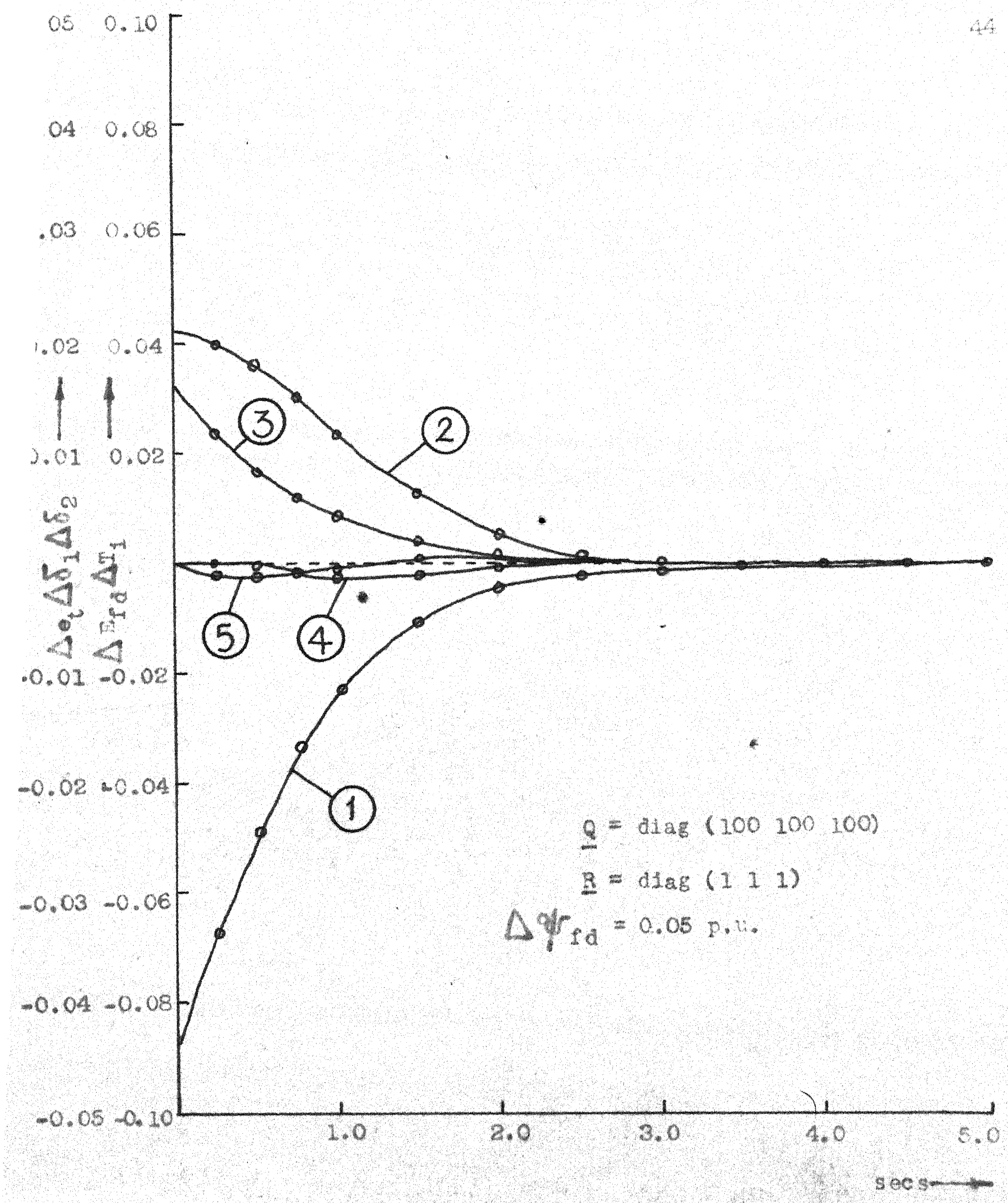


Fig.4.3 Performance of the state regulated optimal system for case (i).

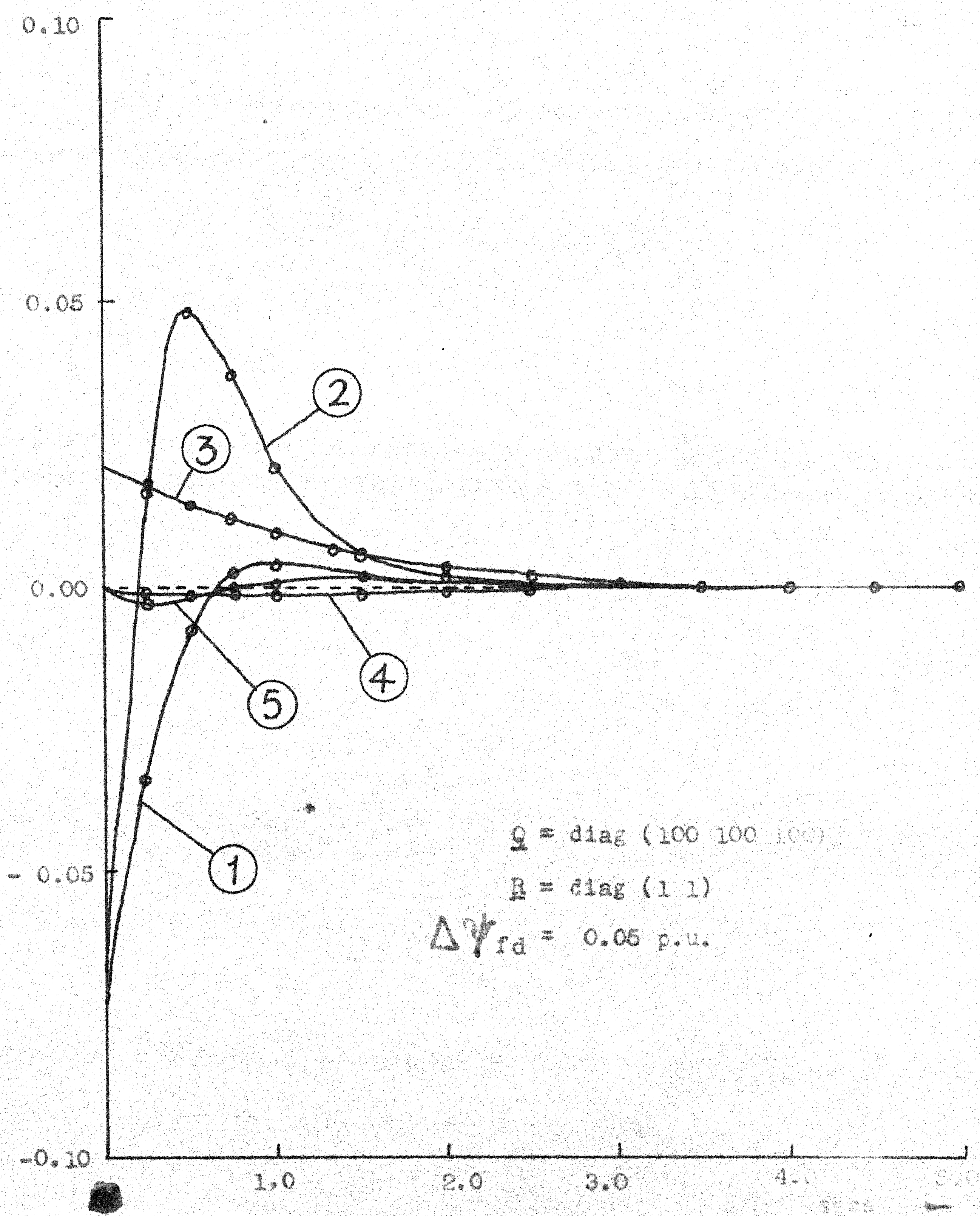


Fig.4.4 Perfomance of the state regulated optimal system for case (ii).

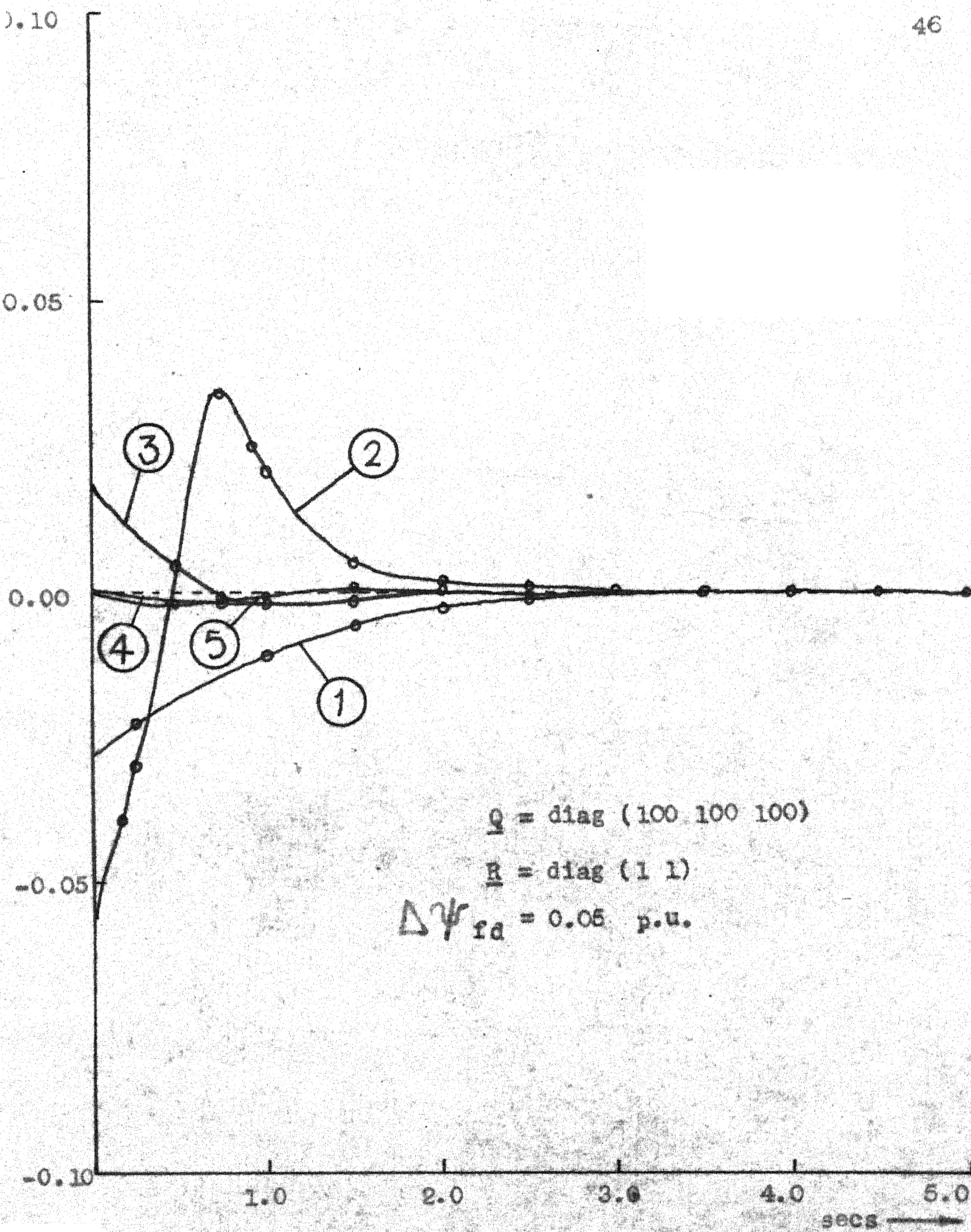


Fig.4.5 Performance of the state regulated optimal system for case (iii).

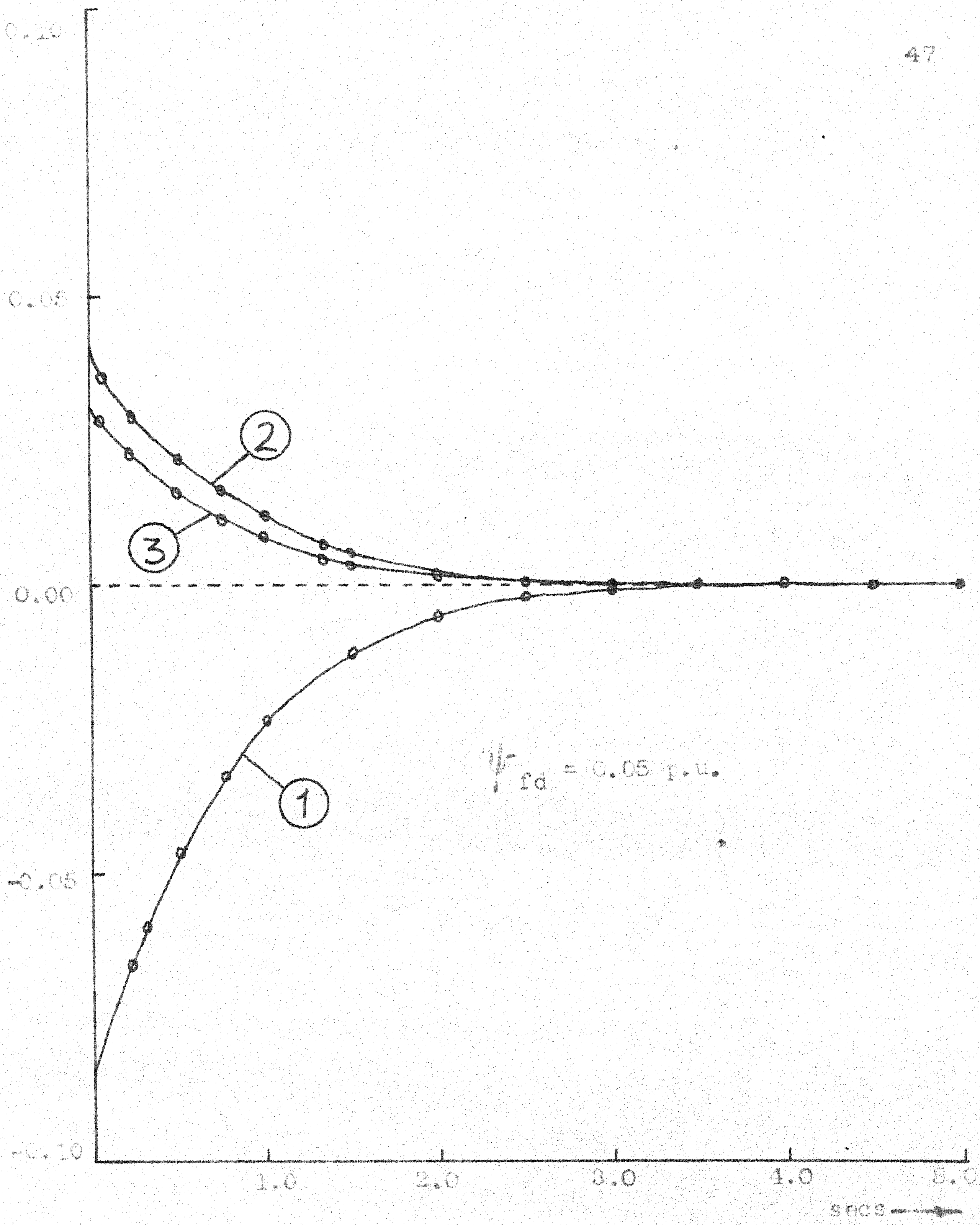


Fig. 4.6 Performance of the nonlinear model with the optimal state regulator.

CHAPTER V

OBSERVER DESIGN FOR THE OPTIMAL OUTPUT REGULATOR

5.1 Introduction:

The design of the optimum output regulator in the previous chapter assumed the availability of all the state variables for the purpose of feedback. It is seldom that all of these variables are accessible for direct measurement. To obviate this difficulty a dynamic compatible observer is designed which reconstructs these inaccessible state variables from the system input and output variables. The reconstructed variables are subject to an exponentially decaying error. Bongiorno and Youla[18] have given a design procedure for the construction of a compatible observer.

The Optimum Control law derived in equation (4.3.6) when cascaded with the observer gives the overall configuration of the optimum regulator.

5.2 Formulation of the Observer Problem:

The observer problem is the reconstruction of the state vector for the system given by equations (4.2.2) and (4.2.3) from the input and output vectors of the system. Before the formal reconstruction is considered, it is worthwhile to investigate whether the system dynamic modes

of behaviour as represented by the state vector can be ascertained from measurements of the available inputs and outputs. The concept of observability is useful for this purpose.

A system is said to be completely observable in the interval $t_0 \leqslant t \leqslant t_1$ for every t_0 and some t_1, if every state $\underline{x}(t)$ can be determined from the knowledge of $\underline{y}(t)$ and $\underline{u}(t)$ in $t_0 \leqslant t \leqslant t_1$. The condition for the system to be completely observable is that the composite matrix

$$\underline{P} = (\underline{C}^T \; \underline{A}^T \; \underline{C}^T \; \ldots\ldots\ldots\ldots \; (\underline{A}^T)^{n-1} \; \underline{C}^T)$$

is of rank n.

To check the observability of the Power System under consideration, the 7x21 matrix $\underline{P}$ is formed for the operating conditions described in Section 2.4, and the determinants of different combinations of 7 columns of matrix $\underline{P}$ were found to be nonzero. Thus the rank of $\underline{P}$ is 7, thereby implying that the system is observable. As a further verification of the result, the system has also been reduced to its normal form, and it was found that the output matrix has no columns which have all zero elements. This ensures that the system is indeed observable[17].

Thus for the linear observable system described by equations (4.2.2) and (4.2.3), the dynamic observer is defined as

$$\underline{\dot{z}} = \underline{F}\,\underline{z} + \underline{G}\,\underline{y} + \underline{H}\,\underline{u}$$

and $$\underline{\hat{x}} = \underline{L}_1\,\underline{y} + \underline{L}_2\,\underline{z} \qquad (5.2.1)$$

where $\underline{F}$, $\underline{G}$, $\underline{H}$, $\underline{L}_1$ and $\underline{L}_2$ are real constant (n-p)x(n-p), (n-p) x p, (n-p)x r, nxp and nx(n-p) matrices respectively, $\underline{\hat{x}}$ is the estimated value of $\underline{x}$ and

$$\underline{z} = \underline{T}\,\underline{x} + \underline{e} \qquad (5.2.2)$$

where $\underline{T}$ is the unique solution of

$$\underline{T}\,\underline{A} - \underline{F}\,\underline{T} = \underline{G}\,\underline{C} \qquad (5.2.3)$$

provided $\underline{A}$ and $\underline{F}$ do not have common eigenvalues. For a choice of

$$\underline{H} = \underline{T}\,\underline{B} \qquad (5.2.4)$$

The error between the actual and observed state vectors at time t is given by

$$\underline{e}(t) = \exp(\underline{F}(t - t_0))\;\underline{e}(t_0) \qquad (5.2.5)$$

and the reconstructed state vector is given by

$$\underline{\hat{x}} = \underline{L}(\underline{W}\,\underline{x} + \underline{\hat{e}}) \qquad (5.2.6)$$

where the vector $\underline{e}$ is defined by

$$\underline{\hat{e}} = \begin{bmatrix} \underline{0}_p \\ \cdots \\ \underline{e} \end{bmatrix} = \begin{bmatrix} \underline{0}_p \\ \cdots\cdots\cdots\cdots \\ \exp(\underline{F}(t - t_0)\;\underline{e}(t_0)) \end{bmatrix} \qquad (5.2.7)$$

In the above equation, $\underline{O}_p$ denotes the p dimensional column vector whose elements are zero, $\underline{F}$ is a stable matrix and $\underline{L}$ and $\underline{W}$ are respectively, the partitioned matrices

$$\left[\underline{L}\right] = \left[\underline{L}_1 \quad \underline{L}_2\right] \tag{5.2.8}$$

and

$$\left[\underline{W}\right] = \begin{bmatrix} \underline{C} \\ \cdots \\ \underline{T} \end{bmatrix} \tag{5.2.9}$$

where
$$\underline{T} = -\phi_o^{-1}(\underline{F})\,\hat{\underline{T}} \tag{5.2.10}$$

$$\hat{\underline{T}} = \underline{\Gamma}_c\,\underline{\Omega}\,\underline{\nabla}_o$$

$$\underline{\Gamma}_c = (\underline{G}\;\underline{F}\,\underline{G}\;\;\underline{F}^2\,\underline{G}\;\;\ldots\;\;\underline{F}^{n-1}\,\underline{G})$$

$$\underline{\nabla}_o = (\underline{C}^T\;\;\underline{A}^T\,\underline{C}^T\;\;(\underline{A}^T)^2\,\underline{C}^T\;\;\ldots\;(\underline{A}^T)^{n-1}\underline{C}^T)^T$$

$$\phi_o\,(\lambda) = \sum_{i=o}^{n} \alpha_i\,\lambda^i \quad \text{with}\;\alpha_n = 1$$

is the characteristic equation of the matrix $\underline{A}$ and α_i's are the co-efficients of the characteristic equation, and

$$\underline{\Omega} = \begin{bmatrix} \alpha_1\,\underline{I}_p & \alpha_2\,\underline{I}_p & \cdots & \alpha_{n-1}\,\underline{I}_p & \alpha_n\,\underline{I}_p \\ \alpha_2\,\underline{I}_p & \alpha_3\,\underline{I}_p & \cdots & \alpha_n\,\underline{I}_p & \underline{O}_p \\ \vdots & \vdots & & \vdots & \vdots \\ \alpha_{n-1}\underline{I}_p & \alpha_n\,\underline{I}_p & \cdots & \underline{O}_p & \underline{O}_p \\ \alpha_n\,\underline{I}_p & \underline{O}_p & \cdots & \underline{O}_p & \underline{O}_p \end{bmatrix}$$

where $\underline{I}_p$ and $\underline{O}_p$ are pxp identify and null matrices respectively.

An observer is said to be compatible if its output equals the state of the plant, to within an exponentially decaying error. A necessary and sufficient condition for an observer to be compatible is the nonsingularity of the matrix $\underline{W}$ given in (5.2.9).

5.3 Construction of an Observer:

The system under consideration requires the construction of seven variables from three outputs and two inputs of the plant. From the procedure outlined in Section 5.2, the dynamics of the observer should be of the fourth order. The matrices $\underline{F}$ and $\underline{G}$ of the compatible observer are so chosen that the pair ($\underline{F}$, $\underline{G}$) represents a controllable system and the matrix $\underline{F}$ should not have any eigenvalues in common with the original system. From the above considerations the $\underline{F}$ and $\underline{G}$ matrices are chosen as follows:

$$\underline{F} = \begin{bmatrix} -7.231 & 0 & 0 & 0 \\ 0 & -7.385 & 0 & 0 \\ 0 & 0 & -7.912 & 0 \\ 0 & 0 & 0 & -7.342 \end{bmatrix} \qquad \underline{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

The matrices $\underline{T}$ and $\underline{W}$ are determined from equations (5.2.10) and (5.2.9) respectively and the matrix $\underline{W}$ is then checked for nonsingularity. The matrices $\underline{H}$ and $\underline{L}$ are obtained from the relations $\underline{H} = \underline{T}\ \ \underline{B}$ and $\underline{L} = \underline{W}^{-1}$. The computation of

matrices $\underline{H}$, $\underline{T}$, $\underline{L}_1$ and $\underline{L}_2$ are done in Double Precision to minimize truncation errors, as the parameters of the observer are highly sensitive, and slight discrepancy leads to improper reconstruction of states. The matrices obtained are

$$\underline{H} = \begin{bmatrix} 0.019505 & 0.000048 \\ -0.000252 & -0.003116 \\ 0.001623 & 0.021459 \\ 0.001886 & 0.023152 \end{bmatrix}$$

$$\underline{T} = \begin{bmatrix} .56x1\bar{0}^{1} & .46x1\bar{0}^{3} & -.38x1\bar{0}^{2} & .13x1\bar{0}^{2} & .14x1\bar{0}^{1} & -.10x1\bar{0}^{2} & .29x1\bar{0}^{3} \\ -.73x1\bar{0}^{3} & .64x1\bar{0}^{5} & .52x1\bar{0}^{4} & -.86x1\bar{0}^{5} & .69x1\bar{0}^{4} & .13497 & -.19x1\bar{0}^{2} \\ .47x1\bar{0}^{2} & -.44x1\bar{0}^{4} & -.38x1\bar{0}^{3} & -.59x1\bar{0}^{4} & -.49x1\bar{0}^{3} & .28x1\bar{0}^{2} & .12875 \\ .55x1\bar{0}^{2} & -.48x1\bar{0}^{4} & -.38x1\bar{0}^{3} & -.64x1\bar{0}^{4} & -.51x1\bar{0}^{3} & .33x1\bar{0}^{2} & .13891 \end{bmatrix}$$

$$\underline{L}_1 = \begin{bmatrix} -0.8295x10^{-1} & 0.3568x10^{7} & -0.1287x10^{3} \\ 0.8454x10 & -0.4529x10^{9} & 0.1427x10^{5} \\ 0.1226x10 & -0.5784x10^{8} & 0.1927x10^{4} \\ 0.1394x10^{2} & -0.6608x10^{9} & 0.2083x10^{5} \\ -0.9027963 & 0.4625x10^{8} & -0.1357x10^{4} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$
\underset{=}{L}_2 = \begin{bmatrix}
-0.1431\times10^{2} & -0.2635\times10^{8} & -0.3205\times10^{6} & -0.3249\times10^{7} \\
0.1846\times10^{4} & 0.3345\times10^{10} & 0.4041\times10^{8} & 0.4127\times10^{9} \\
0.1953\times10^{3} & 0.4272\times10^{9} & 0.5144\times10^{7} & 0.5273\times10^{8} \\
0.2765\times10^{4} & 0.4880\times10^{10} & 0.5897\times10^{8} & 0.6021\times10^{9} \\
-0.1341\times10^{3} & -0.3415\times10^{9} & -0.4122\times10^{7} & -0.4215\times10^{8} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

5.4 Performance of the Optimum Output Feedback Regulator:

Bongiorno and Youla[18] have pointed out that there is an increase in the values of the performance index when a compatible observer is employed to reconstruct the state vector, which in turn is employed as the input to the optimal feedback regulator. Furthermore they have shown that there exists no other choice of the feedback gain matrix which will give a smaller increase in the value of the performance index for all possible initial conditions, when the practical requirement of stability is imposed. Thus the observer designed in Section 5.3 can be introduced into the system which enables the implementation of the optimal control with output feedback. The control is then fed to the unregulated system.

The output response of the system for different initial perturbations of 0.05 p.u. in each of $\Delta\psi_{fd}$, $\Delta\psi_d$, $\Delta\psi_{kd}$, $\Delta\psi_q$, $\Delta\psi_{kq}$, $\Delta\delta_1$, $\Delta\delta_2$ acting one at a time is obtained. Figures (5.1 to 5.7 show the resulting performance curves, for the case when the initial condition vector for the observer is exact. The error introduced in the overall response due to the introduction of the observer is negligible and this is seen by comparing Figure 4.3 with Figure 5.1 for the same initial perturbation in $\Delta\psi_{fd}$=0.5 p.u.

The response due to initial perturbations in $\Delta\psi_{kd}$, $\Delta\psi_q$ and $\Delta\psi_{kq}$ do not have any marked effect on the system performance. This is due to the fact that the time constant associated with the damper coil in the quadrature axis is small. Perturbation in $\Delta\delta_1$ and $\Delta\delta_2$ requires a large controlling effort in ΔT_i as compared to other perturbations, to bring the system to the steady state operating condition quickly. This may be attributed to the inertia of the generator which is to be overcome after the initial perturbation. The perturbation in $\Delta\psi_{fd}$ requires a greater change in field excitation control rather than the input torque, as the field excitation has a major effect on the flux linkage in the field winding. In the case of perturbation in $\Delta\psi_{fd}$ the control effort from both the inputs are almost equal. In all these cases the output

response dies down exponentially and the steady state operating point is reached fairly fast within 2.5 to 3.0 secs. of the initial perturbation in the state variables.

The output response with zero initial conditions for the observer states, for an initial perturbation in $\Delta \psi_{fd}$ = 0.05 p.u. is shown in Fig.5.8. It differs from the output response with exact initial conditions for the observer during the initial stages of the response and takes a slightly longer time to reach the steady state operating condition. Thus the improper choice of the initial conditions for the observer states has an adverse effect during the initial stages of the response only.

The differences in the control law expressions obtained in Section 4.6 for three different operating conditions are relatively small. Thus the same optimal output regulator configuration can be used for all these three operating conditions. However, the same thing cannot be done regarding the observer. The observer is highly sensitive with respect to the changes in the system co-efficient matrices as well as the initial conditions, and therefore the observer designed for one operating condition when used for a different operating condition will have adverse effect on the system response due to the improper

reconstruction of the state. Thus care should be taken to always use a proper observer for each operating condition of the system.

When the optimal routput regulator designed for case (ii) is used along with the observer designed for each specific operating condition, the response for such a combination for the other two operating conditions are shown in Figs.5.9 and 5.10. It is slightly inferior as compared to optimal output regulator and observer for the specific operating condition, thus justifying the use of a common control law.

5.5 Transfer Matrix Representation of the Optimal Output Feedback Regulator:

The observer and the optimal control law cascaded together can be thought of as constituting the Optimal Output Feedback Regulator. The observer discussed in Section 5.2 is given by

$$\underline{\dot{z}} = \underline{F}\ \underline{z} + \underline{G}\ \underline{y} + \underline{H}\ \underline{u}$$

and

$$\underline{\hat{x}} = \underline{L}_1\ \underline{y} + \underline{L}_2\ \underline{z} \qquad (5.4.1)$$

the optimum control law as obtained in Section 4.2 is given by

$$\underline{u}^* = \underline{P}\ \underline{x} \qquad (5.4.2)$$

where

$$\underline{P} = -\underline{R}^{-1}\ \underline{B}^T\ \underline{K}$$

It is observed in Section 5.3 that the effect of assuming zero initial conditions for the observer alters only the initial portion of the response, the error due to improper initial conditions is reasonably small. In view of this it is convenient to assume the initial conditions to be zero. Using this fact, and taking the Laplace Transform of the equations (5.4.1) and (5.4.2) and rearranging

$$(s\,\underline{I} - \underline{F})\,\underline{Z}(s) = \underline{G}\,\underline{Y}(s) + \underline{H}\,\underline{U}(s)$$

or $$Z(s) = (s\,\underline{I} - \underline{F})^{-1}\,(\underline{G}\;\underline{Y}(s) + \underline{H}\,\underline{U}(s))$$

$$\underline{Z}(s) = \underline{D}(s)\,\underline{Y}(s) + \underline{M}(s)\,\underline{U}(s) \qquad (5.4.3)$$

where

$$\underline{D}(s) = (s\,\underline{I} - \underline{F})^{-1}\,\underline{G}$$

and $$\underline{M}(s) = (s\,\underline{I} - \underline{F})^{-1}\,\underline{H}$$

$$\underline{X}(s) = \underline{L}_1\,\underline{Y}(s) + \underline{L}_2\,\underline{Z}(s)$$

and $$\underline{U}(s) = \underline{P}\,\underline{X}(s) \qquad (5.4.4)$$

But $\underline{X}(s) \doteq \hat{\underline{X}}(s)$

$$\underline{U}(s) = \underline{P}(s)\,\underline{L}_1\,\underline{Y}(s) + \underline{P}(s)\,\underline{L}_2\,\underline{D}(s)\,\underline{Y}(s) + \underline{P}(s)\,\underline{L}_2\,\underline{M}(s)\underline{U}(s)$$

or $$\underline{U}(s) = \underline{N}(s)\,\underline{Y}(s) \qquad (5.4.5)$$

where

$$\underline{N}(s) = (\underline{I} - \underline{P}\,\underline{L}_2\,\underline{M}(s))^{-1}\,(\underline{P}\,\underline{L}_1 + \underline{P}\,\underline{L}_2\,\underline{D}(s))$$

In the above equation, the matrices $\underline{P}$, $\underline{L}_1$, $\underline{L}_2$, $\underline{M}(s)$ and $\underline{D}(s)$ are all known and $\underline{N}(s)$ can be calculated.

A typical configuration of the Optimal Output Regulator for the operating condition given in Section 2.4 is shown in Fig.5.11. The implementation of the Optimal Output Regulator thus reduces to the realization of the six elements of the 2x3 transfer matrix $\underline{N}(s)$, shown in Fig.5.11. The outputs of the controller are fed to an exciter to develop the required change in field excitation and to a motor to modify the governor setting to produce the required change in the input torque. Thus, the dynamics of the exciter and the motor must be included in realizing elements of the transfer matrix $\underline{N}(s)$. The rest of the dynamics can be synthesized using appropriate compensating devices. The inputs to the optimal output regulator can be obtained by the following technique. The change in the derivative of the load angle can be obtained as proportional to the difference between a fixed reference and the actual tacho-generator output and integration of the resultant gives the change in load angle. The difference in the terminal voltage and the fixed reference voltage gives the other input.

The outputs of the optimal regulating equipment can be made effective as follows. Change in excitation voltage can be implemented by directly changing the excitation voltage. The other change in the input torque is achieved by controlling the gate opening in the case of a hydro system, or by changing the amount of steam to the turbine in a thermal system.

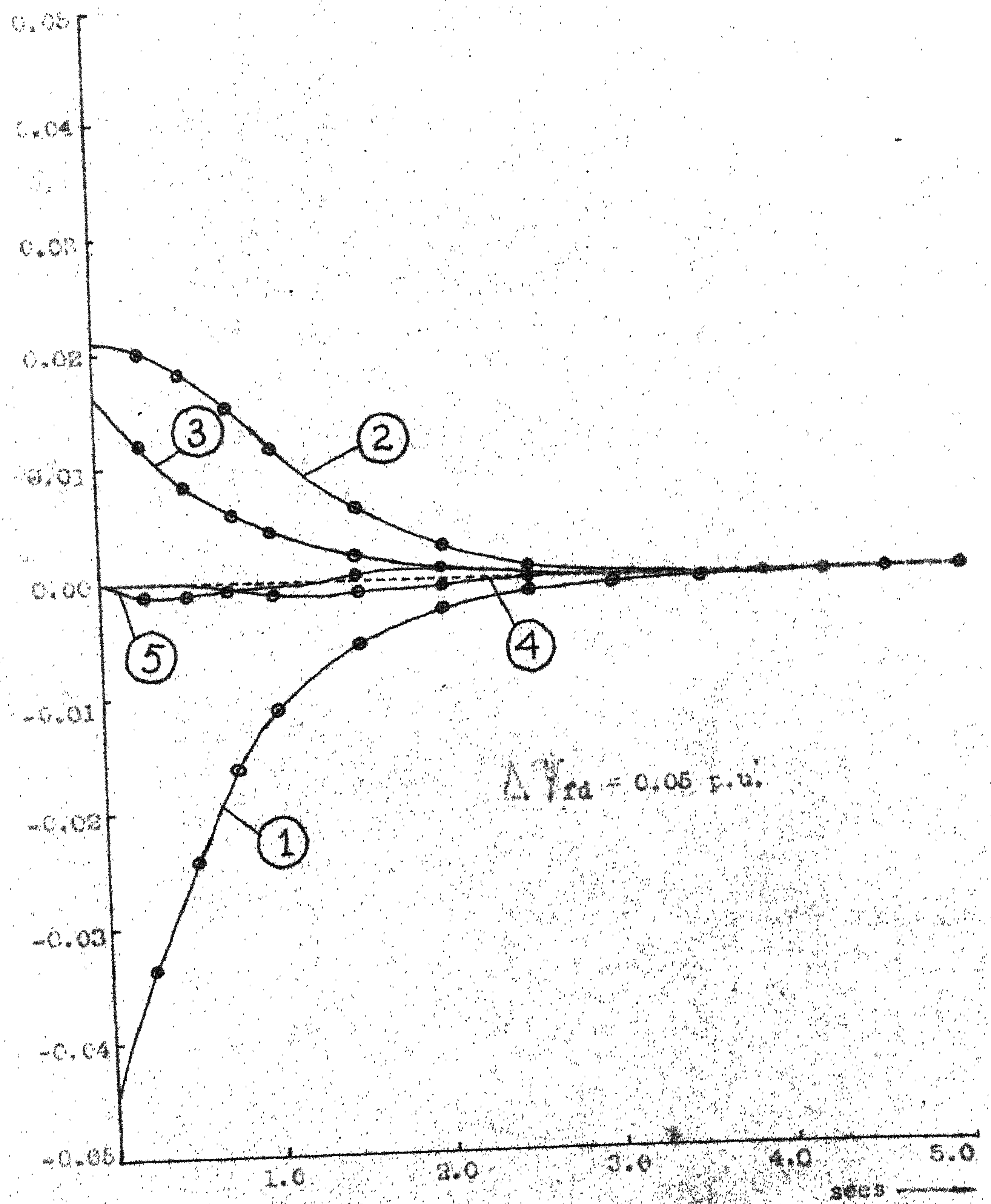


Fig.5.1 Performance of the system with the optimal output feedback regulator for case (i) with exact initial conditions for the observer

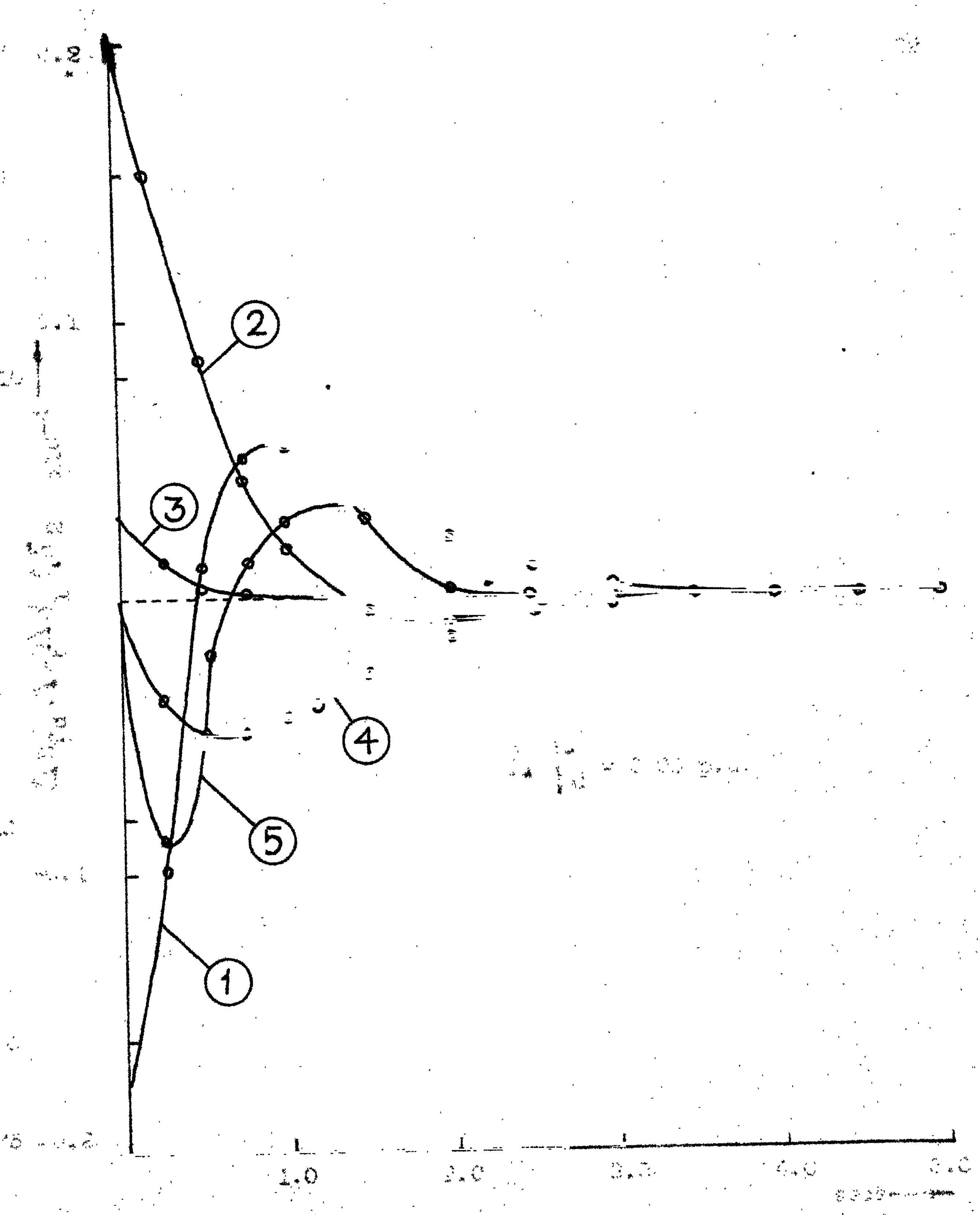


Fig. 5.? Performance of the system with the optimal output feedback regulator for case (1) with exact initial conditions for the observer.

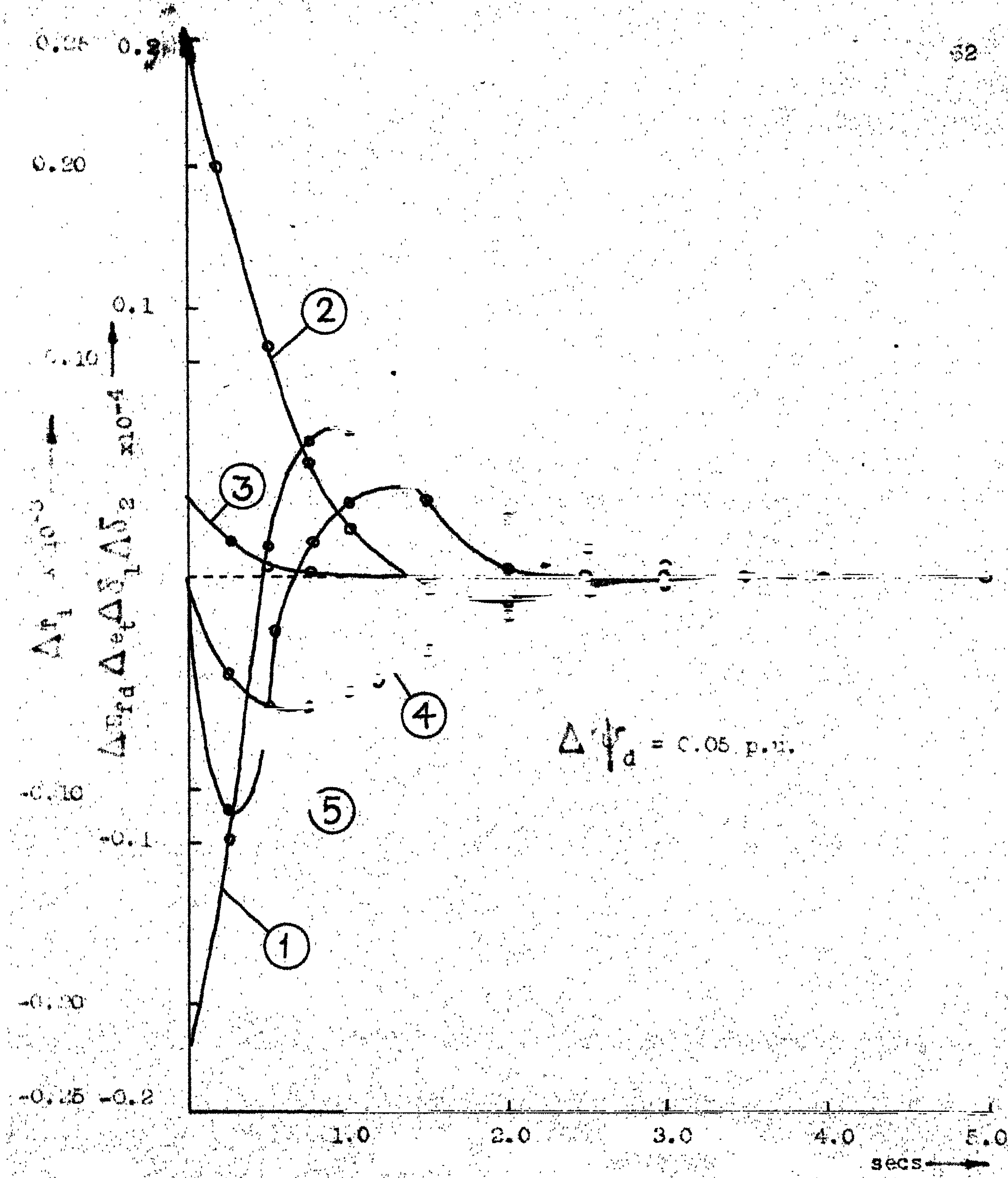


Fig.5.2 Performance of the system with the optimal output feedback regulator for case (i) with exact initial conditions for the observer.

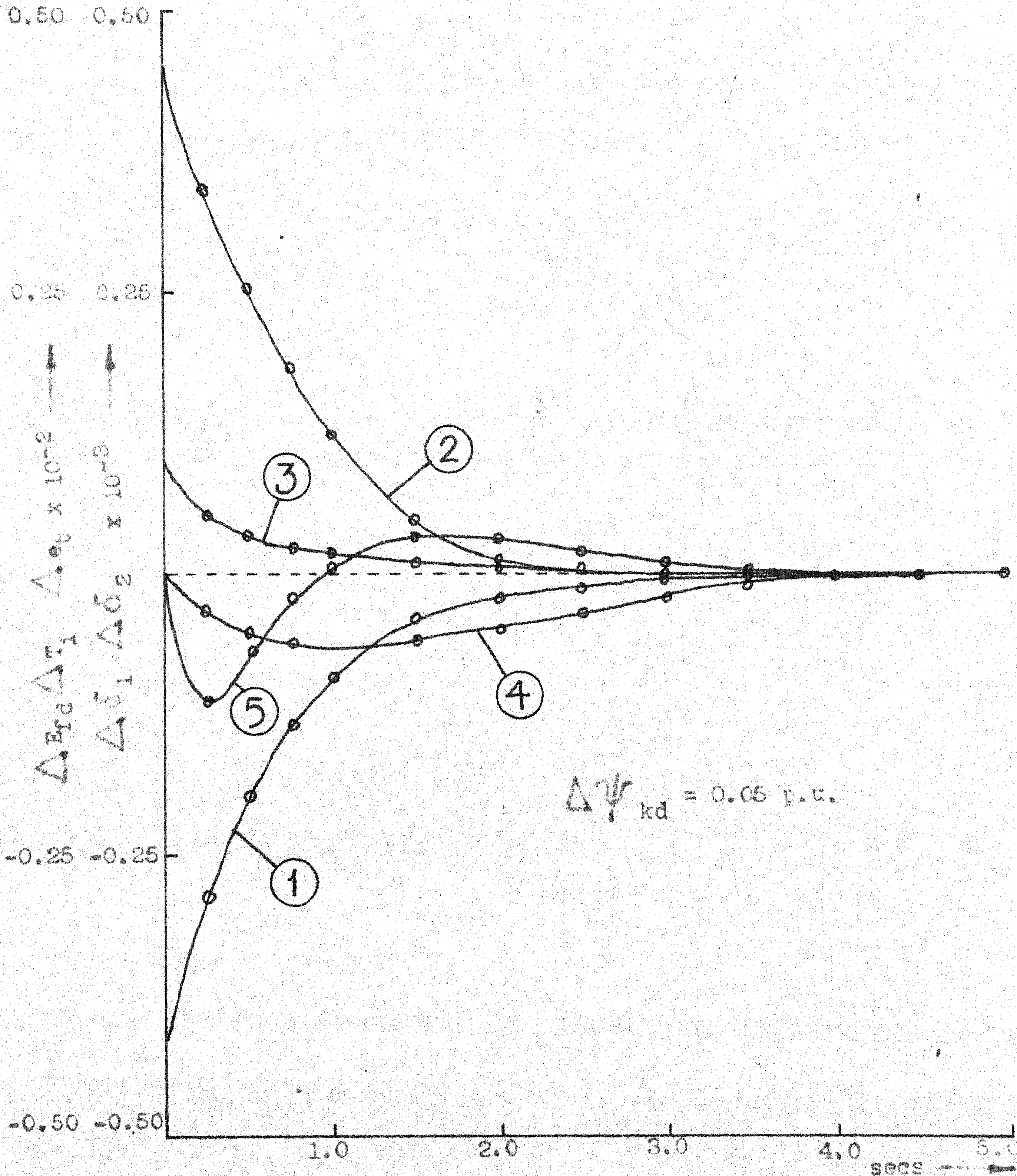


Fig.5.3 Performance of the system with the optimal output feedback regulator for case (i) with exact initial conditions for the observer.

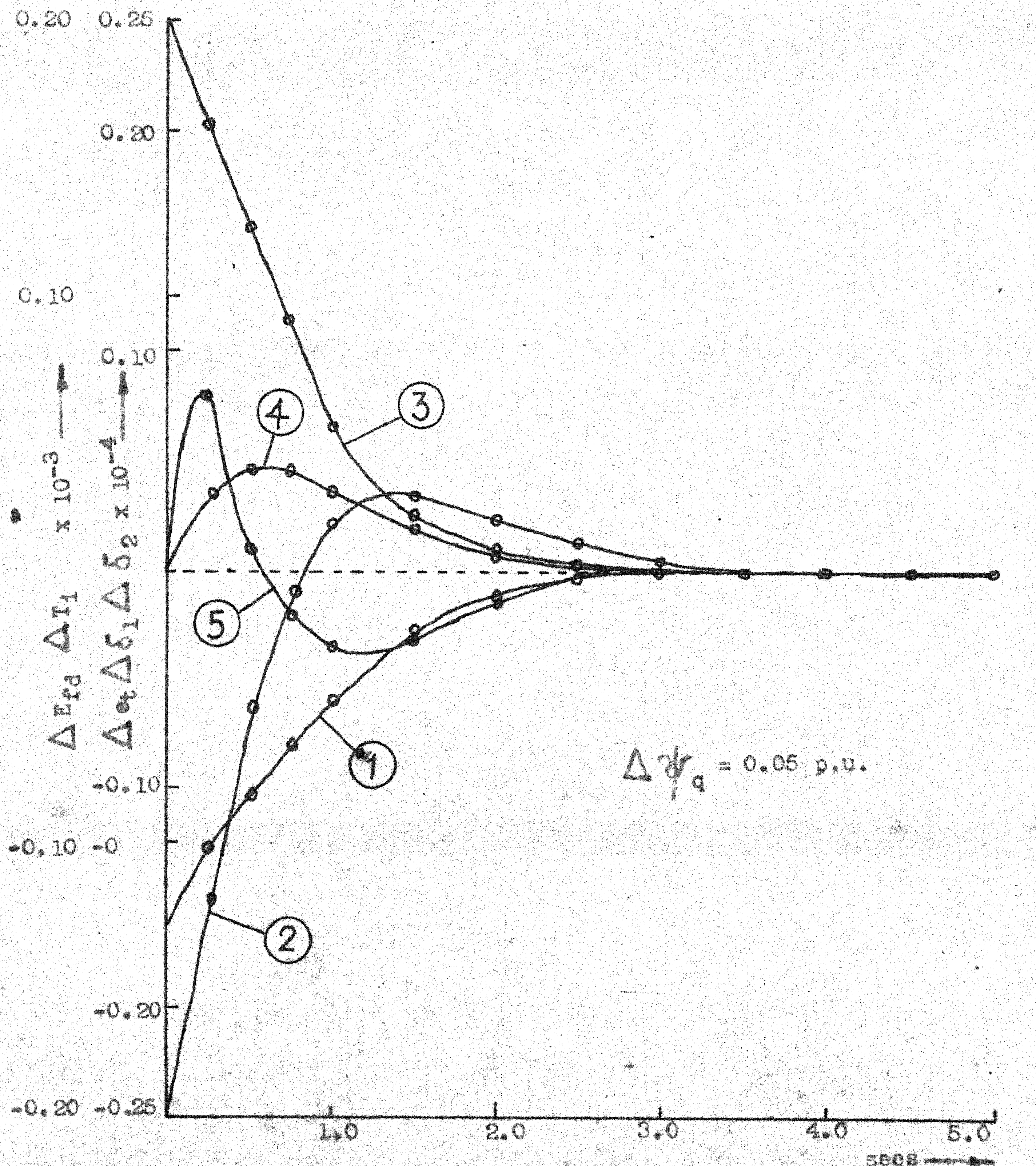


Fig.5.4 Performance of the system with the optimal output feedback regulator for case (i) with exact initial conditions for the observer.

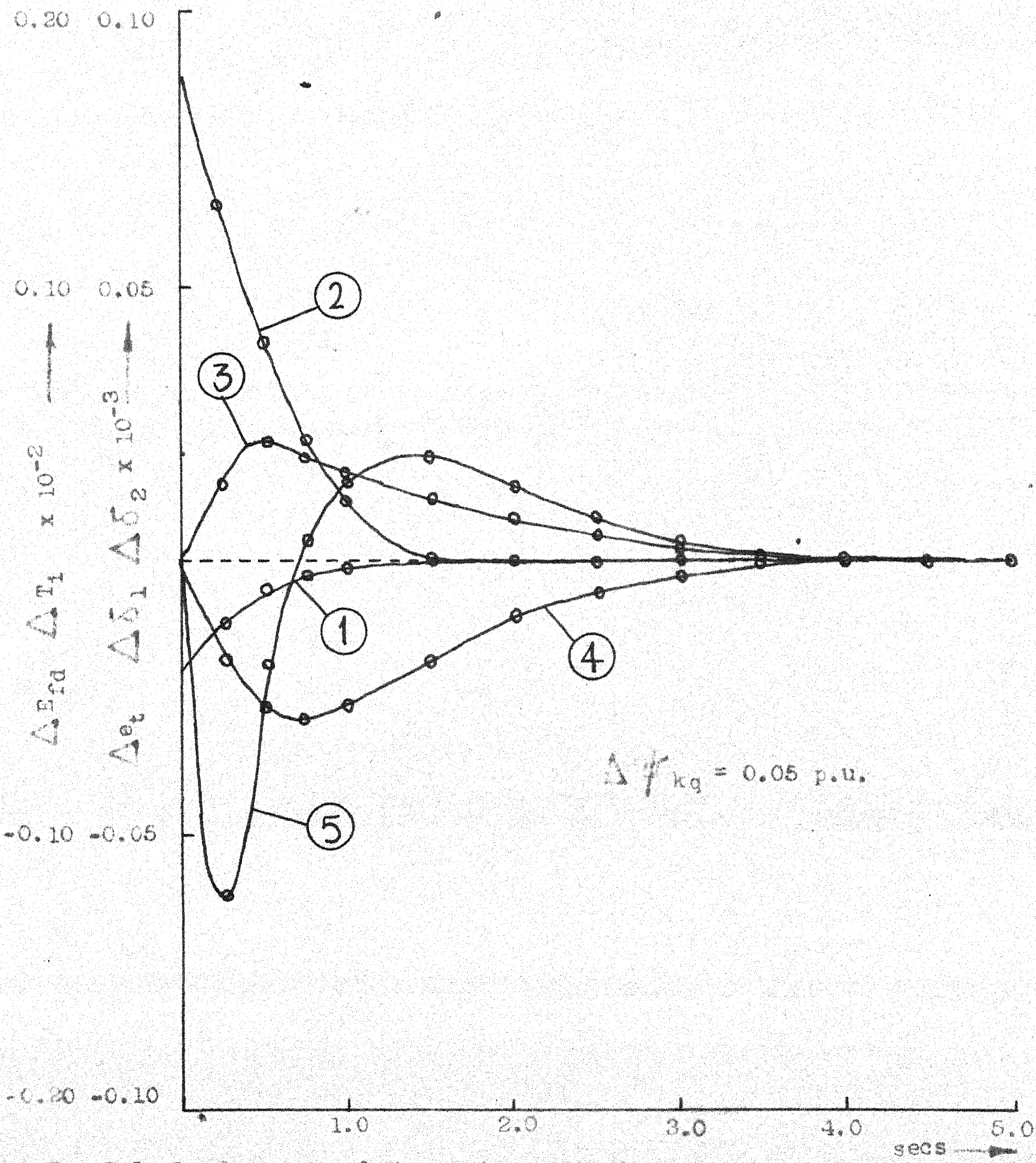


Fig.5.5 Performance of the system with the optimal output feedback regulator for case (i) with exact initial conditions for the observer.

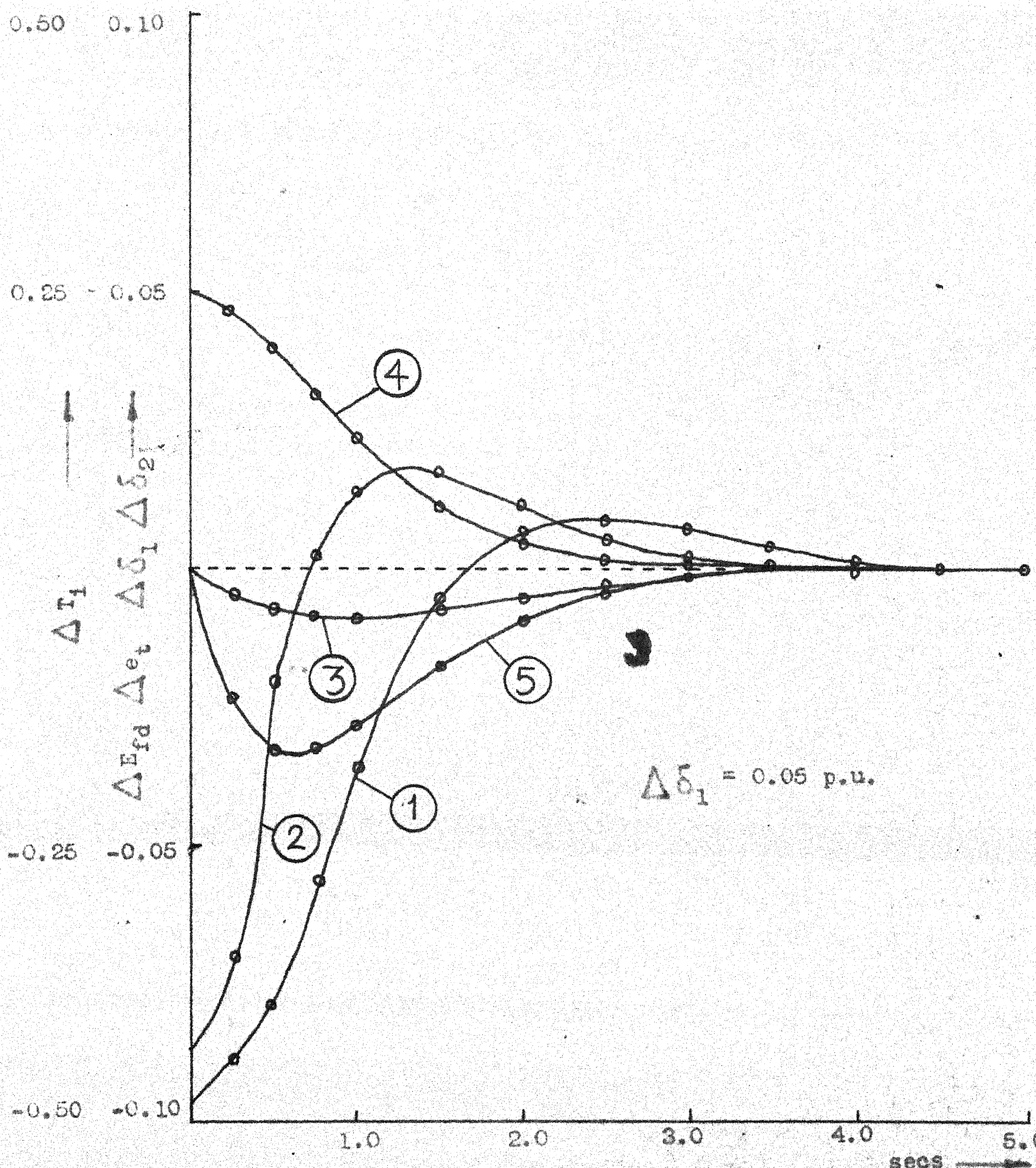


Fig.5.6 Performance of the system with the optimal output feedback regulator for case (i) with exact initial conditions for the observer.

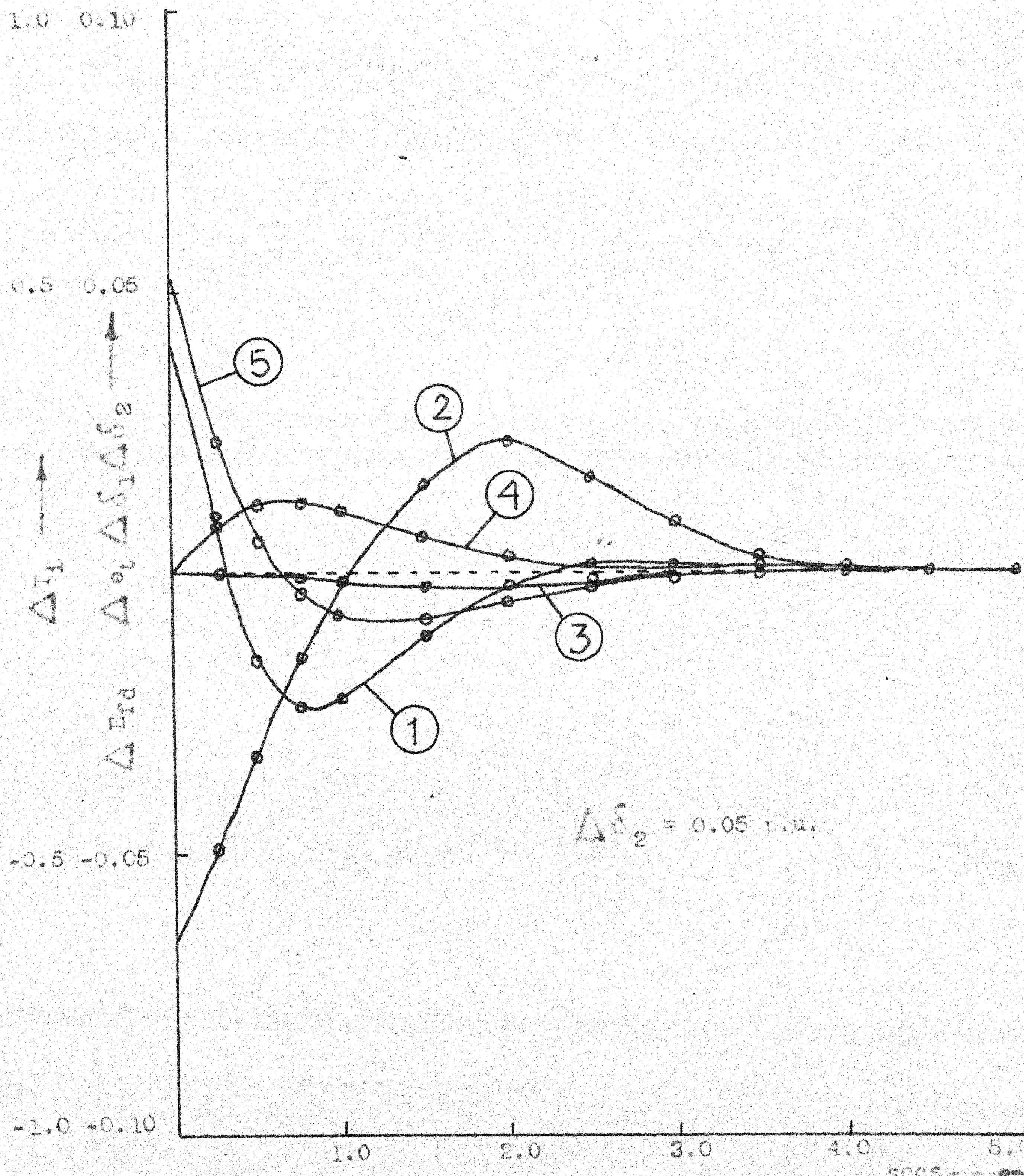


Fig.5.7 Performance of the system with the optimal output feedback regulator for case (i) with exact initial conditions for the observer.

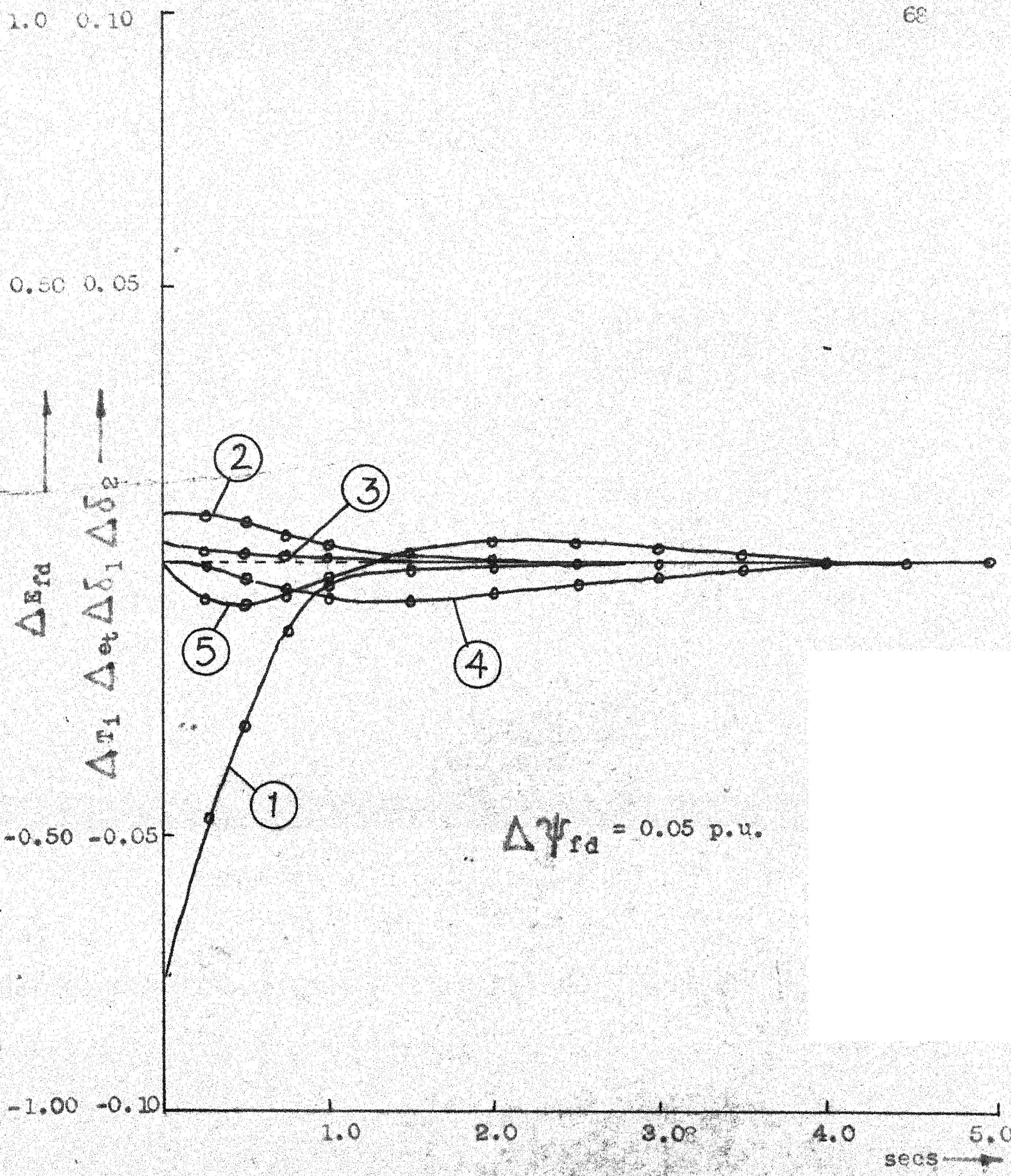


Fig.5.8 Performance of the system with the Optimal output feedback regulator for case (i) with zero initial conditions for the observer.

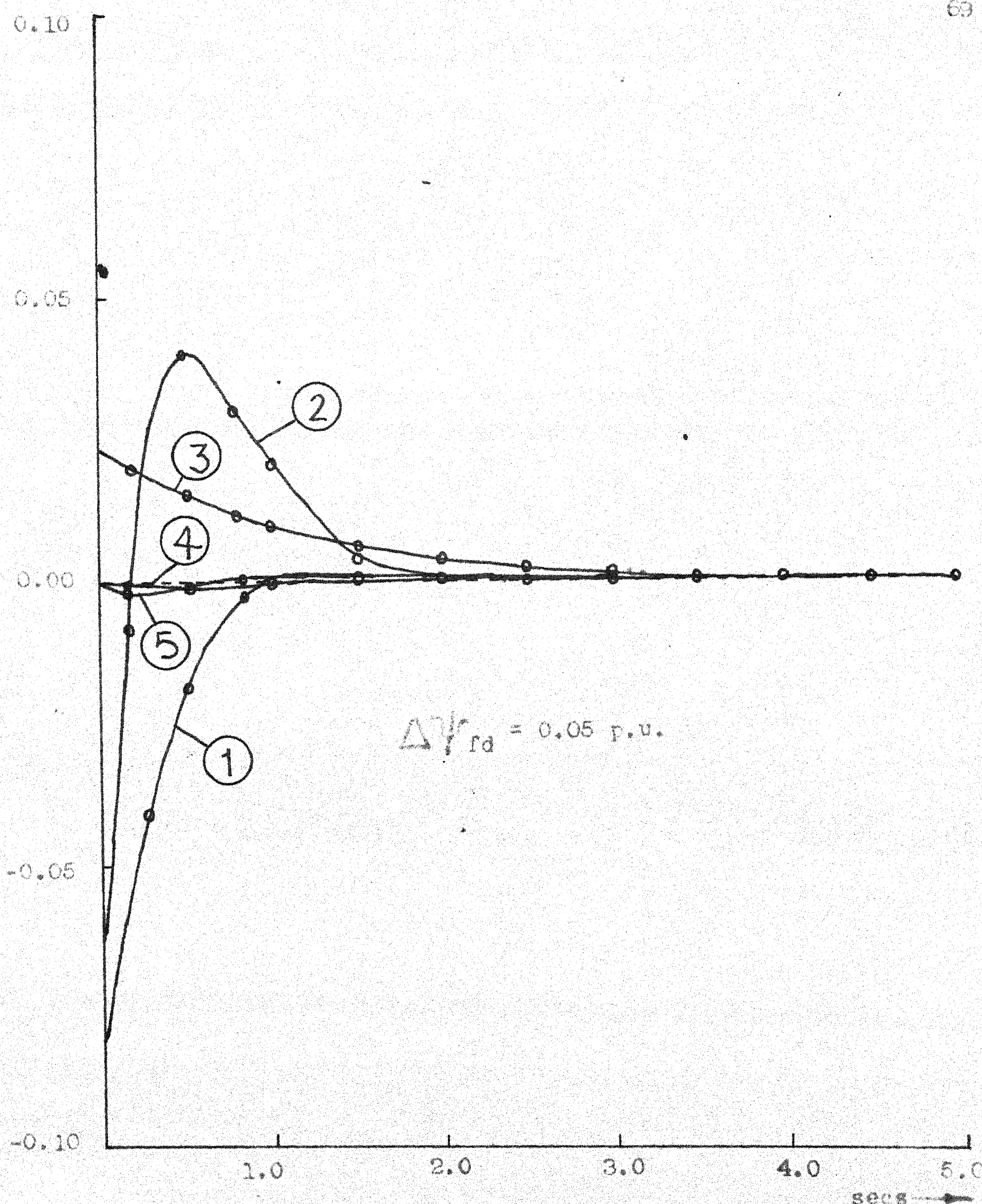


Fig. 5.9 Performance of the system for case (iii) with the optimal control law corresponding to case (ii) with exact initial conditions for the observer.

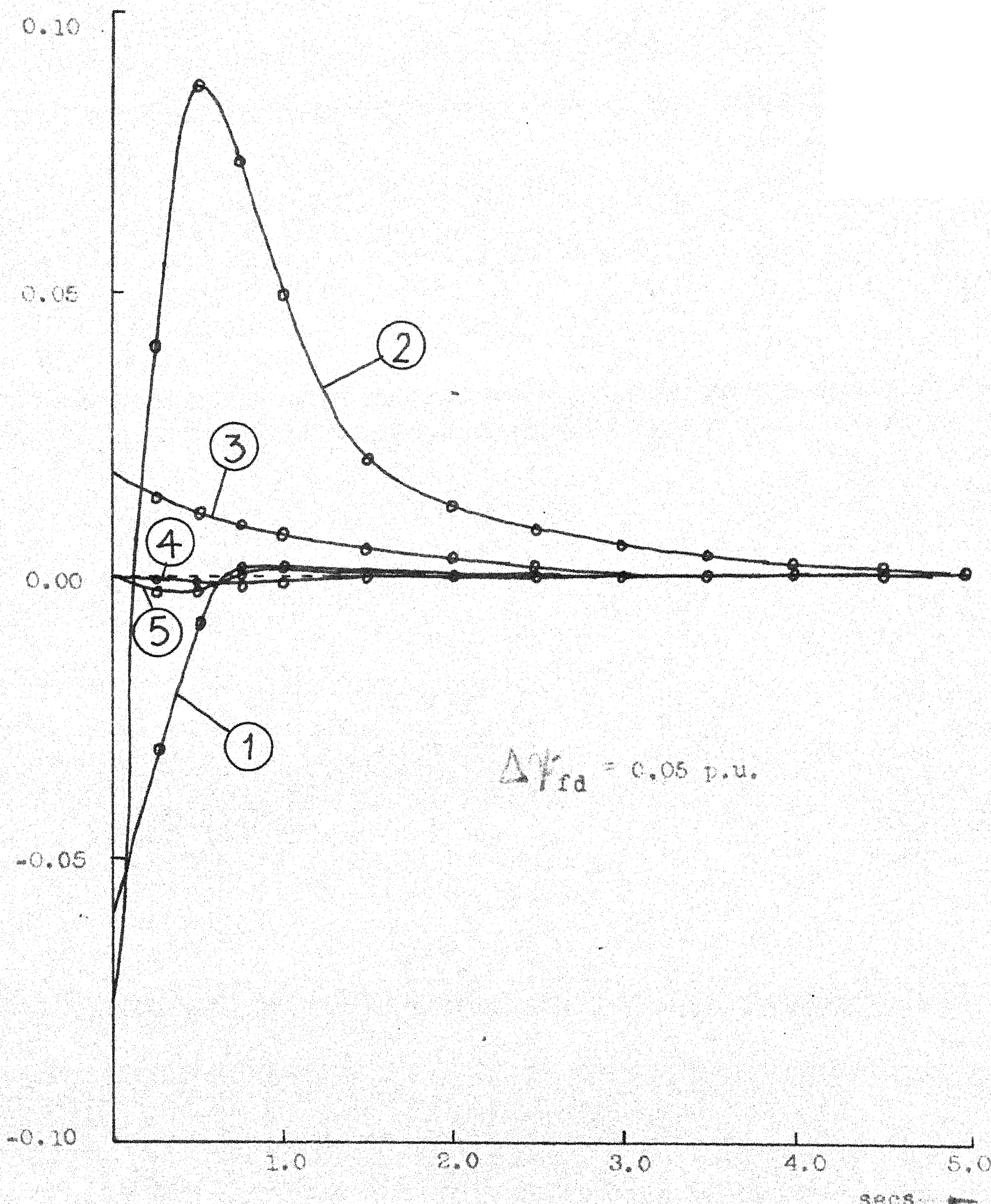


Fig.5.10 Performance of the system for case (i) with the optimal control law corresponding to case (ii) with exact initial conditions for the observer.

may include them as they interact with the generator dynamics. An optimal regulator can be designed for the overall system controlling the prime mover also along with the generator.

(iii) In this thesis the problem formulated is solved as an infinite time output regulator problem. Other possible performance objectives such as minimum time with constraints on the magnitudes of the input variables can also be investigated, although these are difficult to solve analytically. Besides, the minimum time criterion usually results in a bang-bang control, which calls for switching between the extreme values of the permissible control magnitude. Such a large variation in control may render the linearized model chosen for the generator invalid.

(iv) The theory of observers under certain conditions ensures the reconstruction of the state vector within a small error. The design procedure, however, cannot incorporate information regarding the practical aspects of the problem. The observer should have smaller eigenvalues in order that its hardware realization is feasible but this has the drawback that the error in the reconstructed state tends to get large. A suitable compromise has to be made between these conflicting requirements.

(v) In certain situations it is advantageous to use a simplified form of the optimal control law in actual implementation. This simplification can be performed by the method of aggregation suggested by Aoki[19].

APPENDIX A

DERIVATION OF LINEARIZED MODEL

In the derivation of the linearized model the technique of linearization about an operating point is applied, by substituting the perturbed values for the variable quantities in System equations (2.2.29) to (2.2.38). The perturbed equations are:

$$p(\psi_{fdo}+\Delta\psi_{fd}) = \frac{w_o r_{fd}}{x_{f1}}\left(\frac{1}{K_1 x_{f1}}-1\right)(\psi_{fdo}+\Delta\psi_{fd})+\frac{w_o r_{fd}}{K_1 x_{f1} x_{a1}}(\psi_{do}+\Delta\psi_d)$$
$$+\frac{w_o r_{fd}}{K_1 x_{kd1} x_{f1}}(\psi_{dko}+\Delta\psi_{kd}) + \frac{w_o r_{fd}}{x_{ad}}(E_{fdo}+\Delta E_{fd}) \qquad (A.1)$$

$$p(\psi_{do}+\Delta\psi_d) = \frac{w_o(r+e)}{K_1 x_{f1} x_{a1}}(\psi_{fdo}+\Delta\psi_{fd}) + \frac{w_o(r+e)}{x_{a1}}\left(\frac{1}{K_1 x_{a1}}-1\right)(\psi_{do}+\Delta\psi_d)$$
$$+\frac{w_o(r+e)}{K_1 x_{a1} x_{kd1}}(\psi_{kdo}+\Delta\psi_{kd})+\left((w_o+\Delta w)+\frac{w_o f}{x_{a1}}\left(\frac{1}{K_2 x_{a1}}-1\right)\right)(\psi_{qo}+\Delta\psi_q)$$
$$+\frac{w_o f}{K_2 x_{a1} x_{kq1}}(\psi_{kqo}+\Delta\psi_{kq})+ w_o gv\mathrm{Sin}(\delta_{10}+\Delta\delta_1)+w_o hv\mathrm{Cos}(\delta_{10}+\Delta\delta_1) \qquad \ldots\ldots (A.2)$$

$$p(\psi_{kdo}+\Delta\psi_{kd}) = \frac{w_o r_{kd}}{K_1 x_{kd1} x_{f1}}(\psi_{fdo}+\Delta\psi_{fd})+\frac{w_o r_{kd}}{K_1 x_{kd1} x_{a1}}(\psi_{do}+\Delta\psi_d)$$
$$+\frac{w_o r_{kd}}{x_{kd1}}\left(\frac{1}{K_1 x_{kd1}}-1\right)(\psi_{kdo}+\Delta\psi_{kd}) \qquad (A.3)$$

$$p(\psi_{qo}+\Delta\psi_q) = -\frac{w_o f}{K_1 x_{a1} x_{f1}}(\psi_{fdo}+\Delta\psi_{fd})+(-(w_o+\Delta w)\cdot$$

$$+\frac{w_o f}{x_{a1}}(1-\frac{1}{K_1 x_{a1}}))(\psi_{do}+\Delta\psi_d) - \frac{w_o f}{K_1 x_{a1} x_{kd1}}(\psi_{kdo}+\Delta\psi_{kd})$$

$$+\frac{w_o(r+e)}{x_{a1}}(\frac{1}{K_2 x_{a1}}-1)(\psi_{qo}+\Delta\psi_q)+\frac{w_o(r+e)}{K_2 x_{a1} x_{kq1}}(\psi_{kqo}+\Delta\psi_{kq})$$

$$- w_o hv\mathrm{Sin}(\delta_{10}+\Delta\delta_1) + w_o gv\mathrm{Cos}(\delta_{10}+\Delta\delta_1) \qquad (A.4)$$

$$p(\psi_{kqo}+\Delta\psi_{kq}) = \frac{w_o r_{kq}}{K_2 x_{kq1} x_{a1}}(\psi_{qo}+\Delta\psi_q)+\frac{w_o r_{kq}}{x_{q1}}(\frac{1}{K_2 x_{kq1}}-1)(\psi_{kqo}+\Delta\psi_{kq})$$

....(A.5)

$$p(\delta_{10}+\Delta\delta_1) = (\delta_{20}+\Delta\delta_2) \qquad (A.6)$$

$$p(\delta_{20}+\Delta\delta_2) = \frac{1}{T_m}(T_i-K_d(\delta_{20}+\delta_2)-\frac{(\psi_{do}+\Delta\psi_d)(\psi_{qo}+\Delta\psi_q)}{K_2 x_{a1}^2}$$

$$-\frac{(\psi_{do}+\Delta\psi_d)(\psi_{kdo}+\Delta\psi_{kd})}{K_2 x_{a1} x_{kq1}}+\frac{(\psi_{qo}+\Delta\psi_q)(\psi_{do}+\Delta\psi_d)}{K_1 x_{a1}^2}$$

$$+\frac{(\psi_{qo}+\Delta\psi_q)(\psi_{fdo}+\Delta\psi_{fd})}{K_1 x_{a1} x_{f1}}+\frac{(\psi_{qo}+\Delta\psi_q)(\psi_{kdo}+\Delta\psi_{kd})}{K_1 x_{a1} x_{kd1}} \qquad (A.7)$$

$$(e_o+\Delta e_t)^2 = (e_{do}+\Delta e_d)^2 + (e_{qo}+\Delta e_q)^2 \qquad (A.8)$$

$$(\delta_{10}+\Delta\delta_1) = (\delta_{10}+\Delta\delta_1) \qquad (A.9)$$

$$(\delta_{20}+\Delta\delta_2) = (\delta_{20}+\Delta\delta_2) \qquad (A.10)$$

$$(e_{do} + \Delta e_d) = \frac{1}{w_o} p(\psi_{do} + \Delta\psi_d) - \frac{(\psi_{qo} + \Delta\psi_q)(w_o + \Delta w)}{w_o}$$

$$- \frac{r}{x_{a1}} \left(\frac{1}{K_1} \right) \left(\frac{(\psi_{do} + \Delta\psi_d)}{x_{a1}} + \frac{(\psi_{fdo} + \Delta\psi_{fd})}{x_{f1}} + \frac{(\psi_{kdo} + \Delta\psi_{kd})}{x_{a1}} \right)$$

$$- (\psi'_{do} + \Delta\psi_d)) \tag{A.11}$$

$$(e_{qo} + \Delta e_q) = \frac{1}{w_o} p(\psi_{qo} + \Delta\psi_q) + \frac{(\psi_{do} + \Delta\psi_d)(w_o + \Delta w)}{w_o}$$

$$- \frac{r}{x_{a1}} \left(\frac{1}{K_2} \left(\frac{(\psi_{kqo} + \Delta\psi_{kq})}{x_{kq1}} + \frac{(\psi_{qo} + \Delta\psi_q)}{x_{a1}} \right) - (\psi_{qo} + \Delta\psi_q) \right) \tag{A.12}$$

Eliminating the steady state terms defined in equations (2.2.29) to (2.2.38) and retaining first order terms results in the following linearized equations relating the perturbed quantities.

$$p\Delta\psi_{fd} = \frac{w_o r_{fd}}{x_{f1}} \left(\frac{1}{K_1 x_{f1}} - 1 \right) \Delta\psi_{fd} + \frac{w_o r_{fd}}{K_1 x_{f1} x_{a1}} \Delta\psi_d + \frac{w_o r_{fd}}{K_1 x_{kd1} x_{f1}} \Delta\psi_{kd}$$

$$+ \frac{w_o r_{fd}}{x_{ad}} \Delta E_{fd} \tag{A.13}$$

$$p\Delta\psi_d = \frac{w_o(r+e)}{K_1 x_{f1} x_{a1}} \Delta\psi_{fd} + \frac{w_o(r+e)}{x_{a1}} \left(\frac{1}{K_1 x_{a1}} - 1 \right) \Delta\psi_d$$

$$+ \frac{w_o(r+e)}{K_1 x_{a1} x_{kd1}} \Delta\psi_{kd} + \left(w_o + \frac{w_o f}{x_{a1}} \left(\frac{1}{K_2 x_{a1}} - 1 \right) \right) \Delta\psi_q + \frac{w_o f}{K_2 x_{a1} x_{kq1}} \Delta\psi_{kq}$$

$$+ (w_o g v \cos\delta_{10} - w_o h v \sin\delta_{10}) \Delta\delta_1 + \psi_{qo} \Delta\delta_2 \tag{A.14}$$

$$p\Delta\psi_{kd} = \frac{w_o r_{kd}}{K_1 x_{kd1} x_{f1}}\Delta\psi_{fd} + \frac{w_o r_{kd}}{K_1 x_{kd1} x_{a1}}\Delta\psi_d + \frac{w_o r_{kd}}{x_{kd1}}\left(\frac{1}{K_1 x_{kd1}} - 1\right)\Delta\psi_{kd} \quad \ldots.(A.15)$$

$$p\Delta\psi_q = \frac{w_o f}{K_1 x_{a1} x_{f1}}\Delta\psi_{fd} - \left(w_o + \frac{w_o f}{x_{a1}}\left(\frac{1}{K_1 x_{a1}} - 1\right)\right)\Delta\psi_d$$

$$- \frac{w_o f}{K_1 x_{a1} x_{kd1}}\Delta\psi_{kd} + \frac{w_o (r+e)}{x_{a1}}\left(\frac{1}{K_2 x_{a1}} - 1\right)\Delta\psi_q + \frac{w_o (r+e)}{K_2 x_{a1} x_{kq1}}\Delta\psi_{kq}$$

$$- (w_o hv\cos\delta_{10} + w_o gv\sin\delta_{10})\Delta\delta_1 - \psi_{do}\Delta\delta_2 \quad (A.16)$$

$$p\Delta\psi_{kq} = \frac{w_o r_{kq}}{K_2 x_{kq1} x_{a1}}\Delta\psi_q + \frac{w_o r_{kq}}{x_{kq1}}\left(\frac{1}{K_2 x_{kq1}} - 1\right)\Delta\psi_{kq} \quad (A.17)$$

$$p\Delta\delta_1 = \Delta\delta_2 \quad (A.18)$$

$$p\Delta\delta_2 = \frac{\psi_{qo}}{T_m K_1 x_{a1} x_{f1}}\Delta\psi_{fd} + \frac{1}{T_m x_{a1}}\left(\frac{\psi_{qo}}{K_2 x_{a1}} + \frac{\psi_{kqo}}{K_2 x_{kq1}} - \frac{\psi_{qo}}{K_1 x_{a1}}\right)\Delta\psi_d$$

$$+ \frac{\psi_{qo}}{T_m K_1 x_{a1} x_{kd1}}\Delta\psi_{kd} - \frac{1}{T_m x_{a1}}\left(\frac{\psi_{do}}{K_2 x_{a1}} - \frac{\psi_{do}}{K_1 x_{a1}} - \frac{\psi_{fdo}}{K_1 x_{f1}} - \frac{\psi_{kdo}}{K_1 x_{kd1}}\right)\Delta\psi_q$$

$$- \frac{\psi_{do}}{T_m K_2 x_{a1} x_{kq1}}\Delta\psi_{kq} - \frac{K_d}{T_m}\Delta\delta_2 + \frac{\Delta T_i}{T_m} \quad (A.19)$$

Substituting (A.11) and(A.12) in (A.8), and simplifying

$$\Delta e_t = \frac{e_{do}\,e - e_{qo}\,e}{e_o K_1 x_{f1} x_{a1}}\Delta\psi_{fd} + \frac{1}{e_o x_{a1}}\left(\frac{1}{K_1 x_{a1}} - 1\right)(e_{do}e - e_{qo}f)\Delta\psi_d$$

$$+ \frac{(e_{do}e - e_{qo}f)}{e_o K_1 x_{a1} x_{kd1}} \Delta\psi_{kd} + \frac{1}{e_o x_{a1}}\left(\frac{1}{K_2 x_{a1}} - 1\right)(e_{do}f + e_{qo}e)\Delta\psi_q$$

$$+ \frac{1}{e_o}\left(\frac{e_{do}f + e_{qo}e}{K_2 x_{a1} x_{kq1}}\right)\Delta\psi_{kq} + \frac{1}{e_o}(v\cos\delta_{10}(e_{do}g - e_{qo}h)$$

$$- v\sin\delta_{10}(e_{do}h + e_{qo}g)\Delta\delta_1 \qquad \text{(A.20)}$$

$$\Delta\dot{\delta}_1 = \Delta\delta_1 \qquad \text{(A.21)}$$

$$\Delta\dot{\delta}_2 = \Delta\delta_2 \qquad \text{(A.22)}$$

APPENDIX B

RUNGE KUTTA METHOD FOR SOLUTION OF DIFFERENTIAL EQUATIONS

Runge Kutta fourth order method of solving the given system of differential equations is described as follows:

For the system $\frac{d}{dt}(\underline{X}) = \underline{f}(\underline{X}, t)$

$(\underline{X})$ at $t + \Delta t$ can be found from $(\underline{X})$ at t as

$$\underline{X}_{t+\Delta t} = \underline{X}_t + \frac{\underline{K}_1 + 2\underline{K}_2 + 2\underline{K}_3 + \underline{K}_4}{6} \tag{B.1}$$

where

$$\underline{K}_1 = t\ \underline{f}(\underline{X}, t)$$

$$\underline{K}_2 = t\ \underline{f}(\underline{X} + \tfrac{1}{2}\underline{K}_1,\ t + \tfrac{\Delta t}{2})$$

$$\underline{K}_3 = t\ \underline{f}(\underline{X} + \tfrac{1}{2}\underline{K}_2,\ t + \tfrac{\Delta t}{2})$$

$$\underline{K}_4 = t\ \underline{f}(\underline{X} + \underline{K}_3,\ t + \Delta t)$$

The above equation is obtained by using the truncated Taylor series expansion of the function about the initial point $\underline{X}_t$ and equating the corresponding coefficients of $(\Delta t)^n$, where $n = 1,2,3,4$. This method has rounding of errors proportional to $(\Delta t)^5$ and is numerically stable for reasonable values of Δt.

APPENDIX C

The value of the rotor inertia time constant (T_m) of the generator (p.16) is somewhat large. Instead if the value of the inertia constant is teken as 0.019 seconds as taken by Prabhashankar etal[14], while the other data of the system given in Section 2.4 remains unaltered, the following are the corresponding changes:

In equation 2.3.1 on p.36

$$\underline{B} = \begin{bmatrix} 0.3454 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 52.333 \end{bmatrix}^T$$

In equation 2.3.1 on p.37

$$\underline{A} = \begin{bmatrix} -2.1255 & 0.6642 & 1.3285 & 0 & 0 & 0 & 0 \\ 36.231 & -76.085 & 36.231 & 583.14 & -201.86 & 107.25 & -0.6540 \\ 24.154 & 12.077 & -38.646 & 0 & 0 & 0 & 0 \\ 181.150 & -694.42 & 181.15 & -53.829 & 40.371 & -295.12 & -0.5406 \\ 0 & 0 & 0 & 26.914 & -35.886 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -65.819 & 95.438 & -65.819 & 100.87 & -60.624 & 0 & -52.333 \end{bmatrix}$$

In equation 4.3.7 on p.37

$$\underline{K} = \begin{bmatrix} 5.2642 & -0.1394 & 0.3932 & -0.0109 & -0.2213 & 2.7839 & -0.0078 \\ -0.1394 & 0.5027 & -0.1467 & 0.0050 & 0.0025 & 0.6941 & -0.0025 \\ 0.3932 & -0.1467 & 0.1025 & -0.0097 & -0.0742 & 0.2037 & -0.0114 \\ -0.0109 & 0.0050 & -0.0097 & 0.3470 & -0.0967 & 0.0918 & 0.0264 \\ -0.2213 & 0.0025 & -0.0742 & -0.0967 & 0.1597 & -2.7176 & -0.0225 \\ 2.7839 & 0.6941 & 0.2037 & 0.0918 & -2.7176 & 152.90 & 0.2738 \\ -0.0078 & -0.0025 & -0.0114 & 0.0264 & -0.0225 & 0.2738 & 0.1734 \end{bmatrix}$$

In equation 5.2.1 on p. 52

$$\underline{F} = \begin{bmatrix} -7.50 & 0 & 0 & 0 \\ 0 & -7.50 & 0 & 0 \\ 0 & 0 & -7.50 & 0 \\ 0 & 0 & 0 & -7.50 \end{bmatrix} \qquad \underline{G} = \begin{bmatrix} 5.0 & 0 & 5.0 \\ 1.0 & 1.0 & 0 \\ 1.0 & 1.0 & 0 \\ 1.0 & 0 & 1.0 \end{bmatrix}$$

In equation 5.24 on p. 53

$$\underline{H} = \begin{bmatrix} -46.171 & -6.9687 \\ 0.3339 & 0.1827 \\ 0.3339 & 0.1827 \\ -0.9234 & -1.3937 \end{bmatrix}$$

In equation 5.2.10 on p. 53

$$
\underline{T} = \begin{bmatrix}
-1.3367 & .17\times10^{-1} & .98\times10^{-1} & -.13\times10^{-1} & .22411 & -.96604 & -.13316 \\
0.97\times10^{-1} & .74\times10^{-4} & -.71\times10^{-2} & .18\times10^{-2} & .98\times10^{-2} & .15755 & .35\times10^{-2} \\
0.97\times10^{-1} & .74\times10^{-4} & -.71\times10^{-2} & .18\times10^{-2} & .98\times10^{-2} & .15755 & .35\times10^{-2} \\
-.26735 & .33\times10^{-2} & .20\times10^{-1} & -.26\times10^{-2} & .45\times10^{-1} & -.19321 & -.27\times10^{-1}
\end{bmatrix}
$$

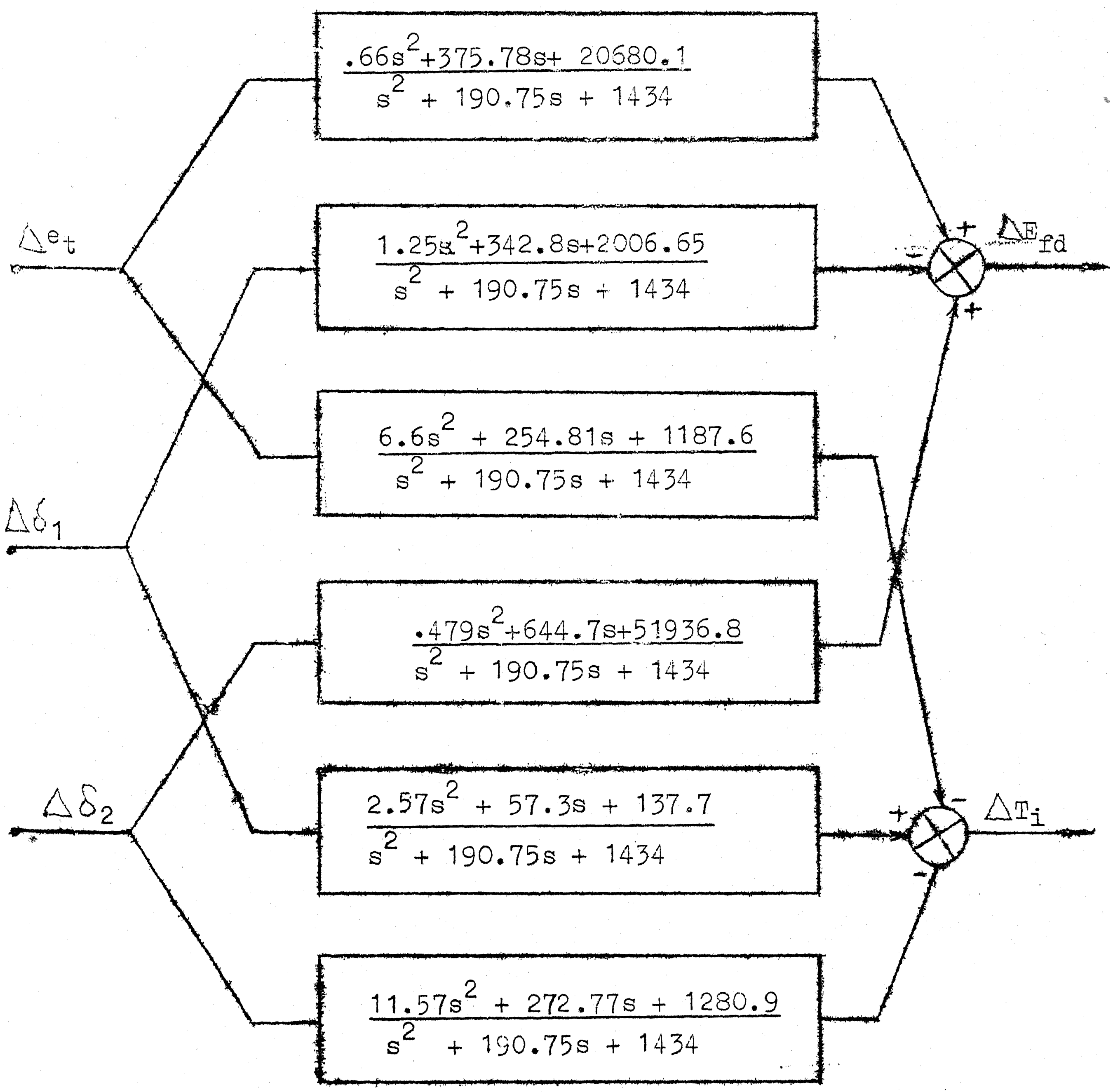
In equation 5.2.8 on p.53

$$
\underline{L}_1 = \begin{bmatrix}
-0.2057 & -0.6036 & -0.1626 \\
0.2274 & -3.4360 & 0.2037 \\
-2.0465 & 7.0544 & -1.1136 \\
3.7533 & -12.112 & 1.4549 \\
-0.1365 & -2.8041 & 0.1778 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

In equation 5.2 on p.54

$$
\underline{L}_2 = \begin{bmatrix}
-0.6567\times10^{15} & 0.1989\times10^{17} & -0.1989\times10^{17} & 0.3284\times10^{16} \\
0.6334\times10^{15} & -0.5797\times10^{17} & 0.5797\times10^{17} & -0.3167\times10^{16} \\
-0.7526\times10^{16} & 0.2190\times10^{18} & -0.2190\times10^{18} & 0.3763\times10^{17} \\
0.7174\times10^{16} & -0.2708\times10^{18} & 0.2708\times10^{18} & -0.3587\times10^{17} \\
-0.2706\times10^{15} & 0.1189\times10^{17} & -0.1189\times10^{17} & 0.1353\times10^{16} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

The transfer matrix $\underline{N}(s)$ defined by equation 5.4.5 on p.71 is



When the exact initial conditions are not known and therefore some arbitrary values are assumed, the choice of the observer dynamics play a key role in determining how fast the initial condition error response decays. For this reason, it is desirable to keep the eigenvalues of $\underline{F}$ matrix reasonably large. On the otherhand, choice of the observer matrices $\underline{F}$ and $\underline{G}$ both play an important role in theoverall configuration of the optimal output regulator. With different choices these matrices so long as the observer has faster dynamics only the initial portion of the overall output response of the system gets affected.

The qualitative performance of the system with the optimal output feedback regulator, with the new inertia time constant, is essentially the same except for an increase in the input torque required during the initial stages of the response. The effect on the Transfer Matrix Representation of the Optimal Output feedback controller is however much more pronounced. The D.C. gains associated with the individual transfer functions are now within a reasonable range as compared to the large values of these gains earlier.

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